Solution to Problem 205E

Section AB of the pipe, with $k_L = k$, has a head loss ΔH_{AB} from point A to point B of:

$$\Delta H_{AB} = \frac{1}{2}\frac{k}{g}u^2.$$

Section EF of the pipe, with again $k_L = k$ has a head loss ΔH_{EF} from point E to point F of:

$$\Delta H_{EF} = \frac{1}{2}\frac{k}{g}u^2.$$

Defining u_{BDE} as the velocity through section BDE of the pipe and u_{BCE} as the velocity through section BCE of the pipe, the head loss from point B to point E can be expressed as:

$$\Delta H_{BE} = \frac{1}{2g}ku_{BDE}^2 = \frac{1}{2g}(4k)u_{BCE}^2$$

and this leads to the following relation between the two velocities u_{BDE} and u_{BCE}:

$$u_{BDE} = 2u_{BCE}$$

As the total volume flow $Q_{AB} = Q_{EF}$ is split into two parts, we know that

$$Q_{BDE} + Q_{BCE} = Q_{AB} = Q_{EF} \quad \Longrightarrow \quad Au_{BDE} + Au_{BCE} = Au$$

and therefore

$$u_{BDE} = \frac{2}{3}u \qquad \text{and} \qquad u_{BCE} = \frac{1}{3}u$$

Substituting these velocities into the expression for the head loss ΔH_{BE}:

$$\Delta H_{BE} = \frac{2}{9}\frac{k}{g}u^2$$

Then by summation, the total head loss in the flow loop, ΔH, is :

$$\Delta H = \Delta H_{AB} + \Delta H_{EF} + \Delta H_{BE} = \frac{11}{9}\frac{k}{g}u^2 = \frac{11}{9}\frac{k}{g}\frac{Q^2}{A^2}$$

where the volume flow rate is defined as $Q = Au$.

The head loss through the flow loop must equal the head rise across the pump, which was given by the pump performance expression

$$\Delta H = \frac{B - CQ^2}{g}$$

Equating this with the expression for the head loss in the flow loop gives:

$$\frac{11}{9}k\frac{Q^2}{A^2} = B - CQ^2$$

which leads to

$$Q^2\left(\frac{11}{9}\frac{k}{A^2} + C\right) = B$$

and solving:

$$Q = \left(\frac{B}{\frac{11}{9}\frac{k}{A^2} + C}\right)^{\frac{1}{2}}$$

The velocity through section BCE then becomes:

$$u_{BCE} = \frac{1}{3}u = \frac{1}{3}\frac{Q}{A}$$

or

$$u_{BCE} = \frac{1}{3A}\left(\frac{B}{\frac{11}{9}\frac{k}{A^2} + C}\right)^{\frac{1}{2}} = \left(\frac{B}{9A^2C + 11k}\right)^{\frac{1}{2}}$$