Solution to Problem 115B

Given that

$$u = U\left(\frac{2y}{ax} - \frac{y^2}{a^2x^2}\right)$$

it follows that

$$\frac{\partial u}{\partial x} = U\left[-\frac{2y}{ax^2} + 2\frac{y^2}{a^2x^3}\right]$$

Now the continuity equation for a 2-D, incompressible flow is

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

and therefore it follows that

$$\frac{\partial v}{\partial y} = U\left[\frac{2y}{ax^2} - 2\frac{y^2}{a^2x^3}\right]$$

Integrating this

$$v(x,y) = \int U\left[\frac{2y}{ax^2} - 2\frac{y^2}{a^2x^3}\right]\partial y = U\left[\frac{y^2}{ax^2} - \frac{2}{3}\frac{y^3}{a^2x^3}\right] + f(x)$$

and since $v(x,0) = f(x) = 0$ it follows that

$$v(x,y) = U\left[\frac{y^2}{ax^2} - \frac{2}{3}\frac{y^3}{a^2x^3}\right]$$