Problem 323A

A long frictionless adiabatic tube of unit cross-sectional area has a steady flow of velocity, u, density, ρ, and temperature, T, of perfect gas flowing in it. The Mach number of this flow is denoted by M and the ratio of the specific heats of the gas is denoted by γ.

At one point along the length of the tube, a very small additional steady flow of gas enters the tube. This additional flow has the same total temperature as the original gas in the tube. Moreover, the injected gas has no momentum in the direction of the axis of the tube. The mass flow rate of injected gas is $\rho u \, di$ where di is a small fraction.

Downstream of the injection point the velocity, pressure, density, etc. have changed by small amounts. Find the small change in the velocity, du, in terms of di, u, M and γ. Neglect terms which are quadratic (or of higher order) in the small quantities, du, di, etc.