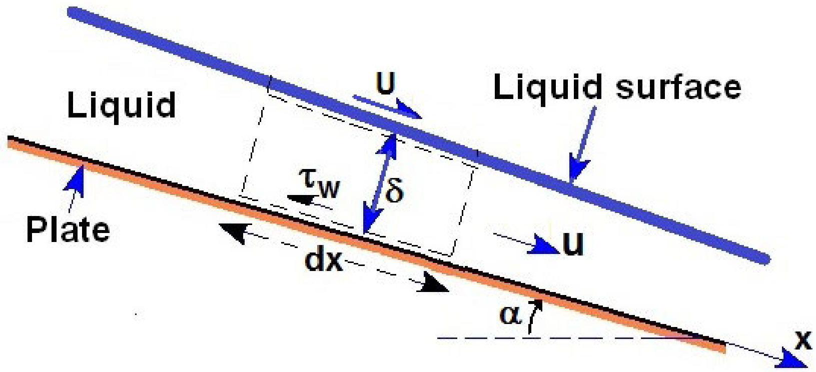
Solution to Problem 452A:

A turbulent flow of liquid emerges from a slot forming a planar flow bounded on top by a free surface and on the bottom by an inclined solid base at an inclination α to the horizontal:



Assuming a velocity profile and wall shear stress similar to that for a turbulent boundary layer on a flat plate:

$$\frac{\overline{u}}{U} = 8.7\left(\frac{y}{\delta}\right)^{1/7} \quad \text{and} \quad \frac{\tau_W}{\rho U^2} = 0.0228\left(\frac{\delta U}{\nu}\right)^{-1/4} \tag{1}$$

Moreover, if the layer thickness, δ, is constant with x it follows from the momentum theorem that the wall shear force per unit length and per unit breadth must balance the weight of the layer per unit length and per unit breadth and therefore

$$\tau_W = \rho g \delta \sin\alpha \tag{2}$$

and therefore the angle that the base must be set at to maintain a constant δ is

$$\alpha = \arcsin\left[\frac{0.0228 U^{7/4}\nu^{1/4}}{g\delta^{5/4}}\right] \tag{3}$$

or

$$\alpha = \arcsin\left[\frac{0.0228 Fr^2}{Re^{1/4}}\right] \tag{4}$$

where the Froude and Reynolds numbers are $Fr = U/(g\delta)^{1/2}$ and $Re = U\delta/\nu$.