Solution to Problem 405A:

A solid surface is characterized as having a nucleation site density distribution function, $N(R)$, where $N(R)dR$ is the number of sites with size between R and $R + dR$ per unit area of the surface. The sites are assumed to have a circular opening of radius, R. If the critical tension is assumed to be $2S/R$ where S is the surface tension and if the distribution function has the form $N(R) = N^*/R^3$ we seek the relation between the superheat and the number of activated nucleation sites per unit surface area.

From the Clausius-Clapeyron relation

$$\frac{\Delta p}{\Delta T} \;=\; \frac{\mathcal{L}\rho_V}{T} \;=\; \frac{2S}{R_{crit}\Delta T} \tag{1}$$

where R_{crit} is the critical size differentiating activated nuclei from non-activated nuclei. Now, if N' is the total number of activated sites per unit surface area it follows that

$$N' \;=\; \int_{R_{crit}}^{\infty} N(R)\; dR \;=\; \int_{R_{crit}}^{\infty} \frac{N^*}{R^3}\; dR \;=\; \frac{1}{2}\frac{N^*}{R_{crit}^2} \tag{2}$$

and therefore

$$N' \;=\; \frac{N^*}{2}\left(\frac{\mathcal{L}\rho_V \Delta T}{2ST}\right)^2 \tag{3}$$