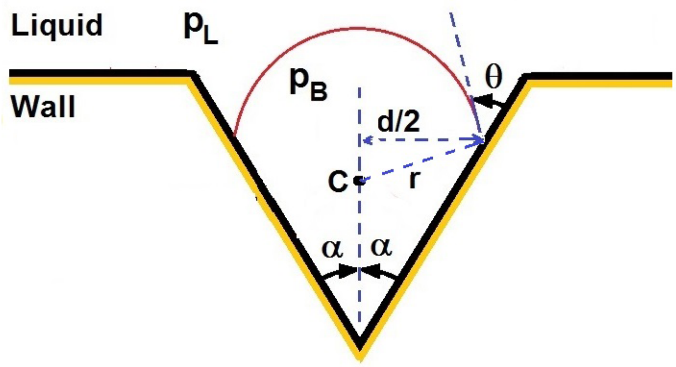
Solution to Problem 403A:

Consider first the hydrophobic case ($\theta > \pi/2$) with the cavity entirely in the crevice The geometry in this



case is such that the bubble radius, r, is related to the diameter of the contact line, d, by

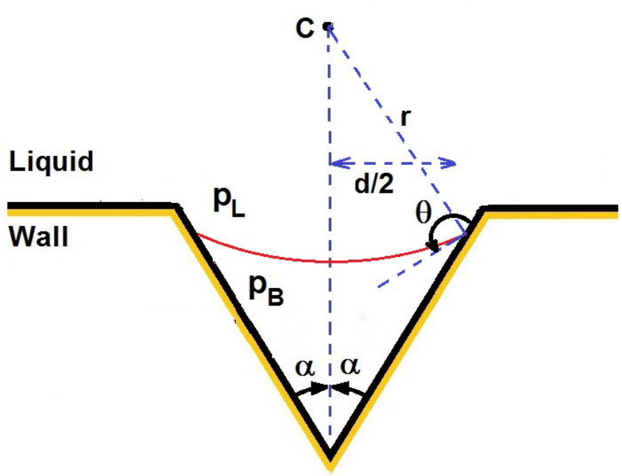
$$\frac{d}{2} = r\cos(\theta - \alpha) \tag{1}$$

and therefore the equilibrium balance of forces across the bubble surface implies that the tension, $(p_B - p_L)$, is

$$p_B - p_L = \frac{2S}{r} = \frac{4S\cos(\theta - \alpha)}{d} \tag{2}$$

Therefore, the deeper the cavity is in the crevice, the smaller are d and r and the larger the tension needs to be to grow the cavity. In other words the smaller d and r are, the larger the tension which the bulk of the liquid can sustain.

On the other hand in the hydrophilic case ($\theta < \pi/2$) with the cavity entirely in the crevice, the curvature of the bubble surface is reversed:



The bubble radius, r, is related to the diameter of the contact line, d, by

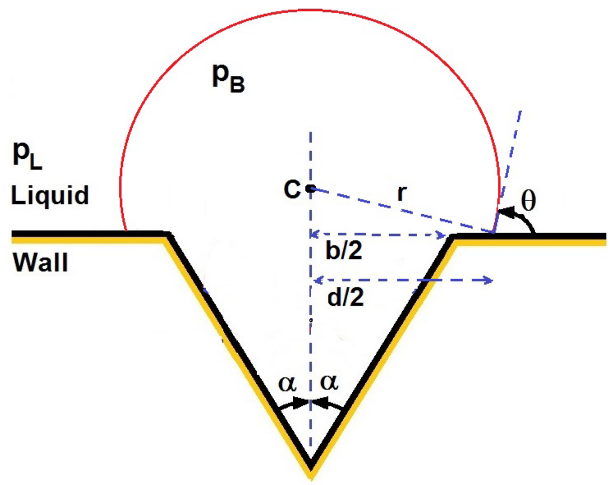
$$\frac{d}{2} = r\cos(\theta - \alpha) \tag{3}$$

and the tension, $p_B - p_L$, becomes

$$p_B - p_L = -\frac{2S}{r} = -\frac{4S\cos(\theta-\alpha)}{d} \tag{4}$$

Consequently when $\theta > \pi/2 + \alpha$ the liquid can sustain a small equilibrium tension but when the surface is very wetting and $\theta < \pi/2 + \alpha$ no tension can be sustained and the bubble will expand out of the crevice.

When the bubble becomes large enough to emerge from the crevice as shown below:



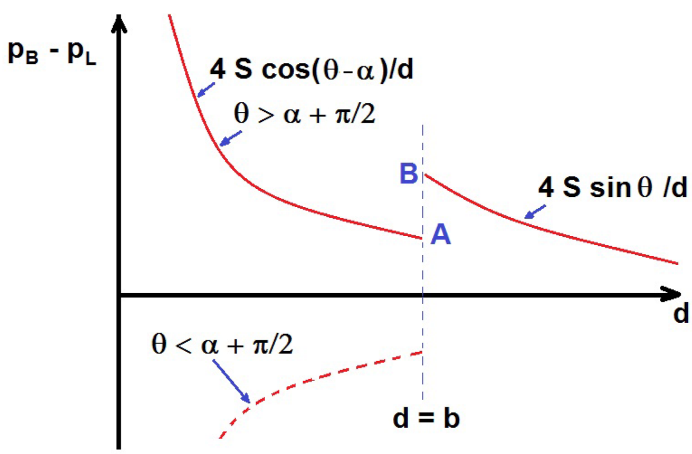
the bubble radius is related to the bubble footprint diameter, d, by

$$\frac{d}{2} = r\sin\theta \tag{5}$$

and the equilibrium tension becomes

$$p_B - p_L = \frac{2S}{r} = \frac{4S\sin\theta}{d} \tag{6}$$

In summary we may plot the possible equilibrium tension, $(p_B - p_L)$, against the bubble base diameter, d:



Note the discontinuity as the bubble transitions out of the crevice. The point B is above the point A when $\theta > \alpha/2 + \pi/4$ which would usually be true in the hydrophilic case. In such a circumstance one can envisage that under these circumstances a growing bubble would become stationary at the crevice exit. On the other had when $\theta > \alpha/2 + \pi/4$ the point B would be below the point A and a growing bubble would pop out of the crevice.