Solution to Problem 352B

Region 2: Prandtl-Meyer Expansion

We calculate the Mach number in region 2 by determining the value of the Prandtl-Meyer function in this region. We use the chart of tabulated Prandtl-Meyer function values to get ν_1.

$$\nu_1(M_1 = 3) = 49.76^\circ$$

The value of the Prandtl-Meyer function in region 2 is then the value in region 1 plus the turn angle.

$$\nu_2 = 49.76^\circ + 20^\circ = 69.76^\circ$$

Using the chart again to get the Mach number in region 2:

$$\Rightarrow M_2 = 4.32$$

Since the expansion is an isentropic process, we can use the isentropic flow relations to find the pressure ratio between regions 1 and 2.

$$\frac{p_0}{p} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{\gamma}{\gamma-1}}$$

$$\frac{p_2}{p_1} = \frac{p_2}{p_0}\frac{p_0}{p_1} = \left(\frac{1 + \frac{\gamma-1}{2}M_1^2}{1 + \frac{\gamma-1}{2}M_2^2}\right)^{\frac{\gamma}{\gamma-1}} = 0.1593$$

Region 3: Oblique Shock

From the graph of oblique shock properties with $M_1 = 3$, $\theta = 20^\circ$:

$$\beta = 37.8^\circ$$

The incoming Mach number normal to the oblique shock is then:

$$M_{n1} = M_1 \sin\beta = 3 \sin 37.8^\circ = 1.84$$

We can then use the normal shock relations to find the pressure ratio across the oblique shock.

$$\frac{p_3}{p_1} = 1 + \frac{2\gamma}{\gamma + 1}\left(M_{1n}^2 - 1\right) = 3.78$$

Forces

Calculating the lift as the pressure over the area, A, on each surface of the flat plate airfoil projected onto the direction perpendicular to the oncoming freestream.

$$L = p_3 A \cos 20^\circ - p_2 A \cos 20^\circ$$

$$C_L = \frac{L}{\frac{1}{2}\rho_1 U_1^2 A} = \frac{p_3 - p_2}{\frac{1}{2}\rho_1 U_1^2}\cos 20^\circ$$

Using the definition of the sound speed ($a^2 = \frac{\gamma p}{\rho}$) to write the coefficient of lift in terms of the Mach number:

$$C_L = \left(\frac{p_3}{p_1} - \frac{p_2}{p_1}\right)\frac{2}{\gamma M_1^2}\cos 20^\circ = 0.540$$

Comparing this to the result from the theory for small angles of turn:

$$C_L = \frac{4\alpha}{\sqrt{M_1^2 - 1}} = 0.494$$