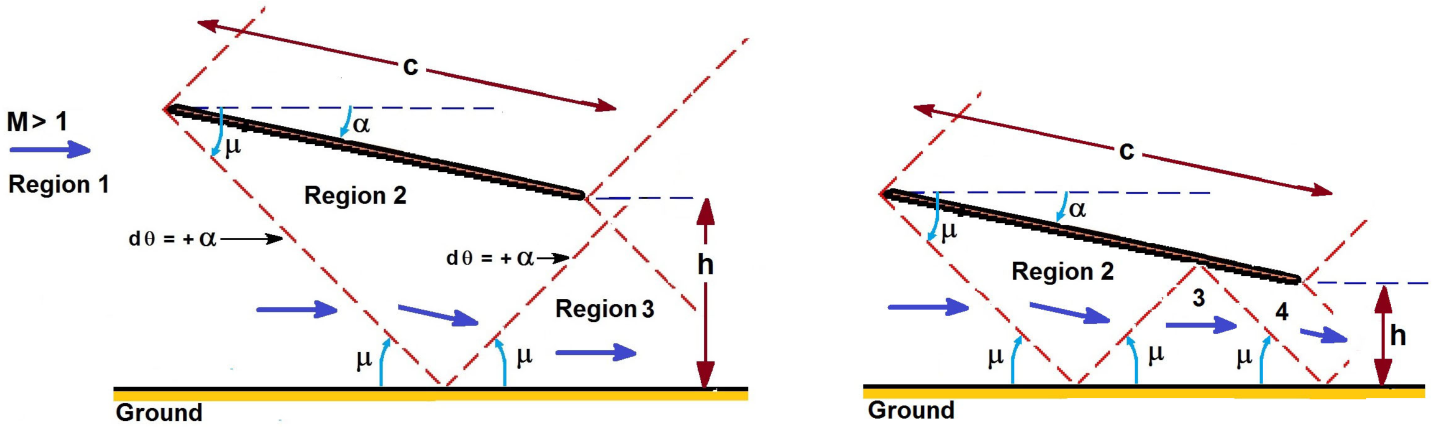
Solution to Problem 340F:

Since the angle of attack, α, is small and the reflected Mach wave, like the leading edge Mach wave, is inclined at $\mu = \arcsin(1/M)$ to the wall, it follows that



(a) If c is equal to $2h/\tan\mu$ then the Mach wave reflected by the wall will precisely impinge on the trailing edge if

$$\left(\frac{h}{c}\right)_{groundeffectbegins} = \frac{\tan\mu}{2} = \frac{1}{2\sqrt{(M^2-1)}} = 0.447 \tag{1}$$

When h/c is greater than 0.447 there is no ground effect on the foil. When h/c is less than this the lift on the foil is greater because of the increased pressure on the last section of the pressure surface.

(b) If $h/c = 0.2$ the Mach wave below the foil will reflect twice from the foil before escaping past the trailing edge as shown on the right above. Denoting the angle of attack by α the compression from region 1 to region 2 will result in

$$p_2 = p_1 + \frac{\rho_1 q_1^2 \alpha}{(M^2-1)^{1/2}} \tag{2}$$

Further compression through the first wave reflected from the wall leads to

$$p_3 = p_1 + \frac{2\rho_1 q_1^2 \alpha}{(M^2-1)^{1/2}} \tag{3}$$

and moreover

$$p_4 = p_1 + \frac{3\rho_1 q_1^2 \alpha}{(M^2-1)^{1/2}} \tag{4}$$

$$p_5 = p_1 + \frac{4\rho_1 q_1^2 \alpha}{(M^2-1)^{1/2}} \tag{5}$$

$$p_6 = p_1 + \frac{5\rho_1 q_1^2 \alpha}{(M^2-1)^{1/2}} \tag{6}$$

On the other hand the pressure over the entire suction surface is

$$p_1 - \frac{\rho_1 q_1^2 \alpha}{(M^2-1)^{1/2}} \tag{7}$$

and so the pressure difference across the foil in the absence of the wall is

$$\frac{2\rho_1 q_1^2 \alpha}{(M^2-1)^{1/2}} \tag{8}$$

and the corresponding lift force per unit depth normal to the sketch is

$$\frac{2\rho_1 q_1^2 c\alpha}{(M^2-1)^{1/2}} \tag{9}$$

which leads to a lift coefficient

$$C_L \;=\; \frac{4\alpha}{(M^2-1)^{1/2}} \tag{10}$$

Now, when the lift effect is included by including the region 4 (and later region 6) in the above diagram the total lift per unit depth normal to the sketch when the downstream end of region 4 just impinges on the trailing edge (and $h/c = 0.247$) becomes

$$\frac{2\rho_1 q_1^2 c\alpha}{(M^2-1)^{1/2}}\left[\frac{1}{2}+\frac{2}{2}\right] \tag{11}$$

which leads to a lift coefficient

$$C_L \;=\; 1.5\times\frac{4\alpha}{(M^2-1)^{1/2}} \tag{12}$$

Then the lift with the ground effect is 1.5 times the lift without the ground effect.

Going one step further, if $h/c = 0.149$, then the Mach wave emanating from the leading edge bounces off the wall three times before impacting the trailing edge and the lift per unit depth normal to the sketch becomes

$$\frac{2\rho_1 q_1^2 c\alpha}{(M^2-1)^{1/2}}\left[\frac{1}{3}+\frac{2}{3}+\frac{3}{3}\right] \tag{13}$$

which leads to a lift coefficient

$$C_L \;=\; 2\times\frac{4\alpha}{(M^2-1)^{1/2}} \tag{14}$$

Then the lift with the ground effect is twice the lift without the ground effect.