Solution to Problem 336A:

Compressed air ($\gamma = 1.4$) is supplied from a reservoir to a pipe, $1.0cm$ in diameter and $5.0m$ long. It is estimated that the average friction factor, f, of the flow in the pipe is 0.02. At the end of this long pipe is a short nozzle whose opening to the atmosphere has one half of the cross-sectional area of the pipe. Assuming that frictional effects in the nozzle can be neglected, we seek the following information pertaining to conditions when the flow through the pipe/nozzle combination is choked.

[A] **In the nozzle:** Neglecting frictional effects in the nozzle, choked flow occurs with $A/A^* = 2$ where A is the entrance area and A^* is the exit area. Then the flow at the end of the pipe (or entrance to the nozzle, point 2) has $M_2 = 0.31$ and $p_2/p_{02} = 0.936$. But since $p^*/p_0 = 0.528$ it follows that

$$\frac{\text{Pressure at 2, the entrance to the nozzle}}{\text{Nozzle exit pressure, } p^*} = \frac{0.936}{0.528} = 1.77 \tag{1}$$

[B] **In the long pipe:** If the pipe continued beyond the point 2, then the distance L_2 from the point 2 to the point where it hypothetically would reach $M = 1$ is given from the table on frictional effects in compressible pipe flow (with an entrance Mach number of 0.31) by

$$\frac{fL_2}{D} = 4.93 \quad \text{and} \quad \frac{p_2}{p^*} = 3.5 \tag{2}$$

But with the actual length of $L = 5m$ it follows that

$$\frac{fL}{D} = \frac{0.02 \times 5}{0.01} = 10 \tag{3}$$

Therefore the distance L_1 from the pipe entrance to the hypothetical $M = 1$ point is given by

$$\frac{fL_1}{D} = 10 + 4.93 = 14.93 \tag{4}$$

and from the frictional table when $fL_1/D = 14.93$ and the pipe entrance Mach number of $M = 0.2$ it follows that $p_1/p_{01} = 0.972$. Therefore

$$\frac{\text{Reservoir pressure}}{\text{Nozzle throat pressure}} = \frac{p_{01}}{p_1}\frac{p_1}{p_2}\frac{p_2}{p_{nozzlethroat}} = \frac{1.57 \times 1.77}{0.972} = 2.86 \tag{5}$$

Therefore the ratio of the pressure in the reservoir to the pressure in the exit (throat) from the nozzle is 2.86.