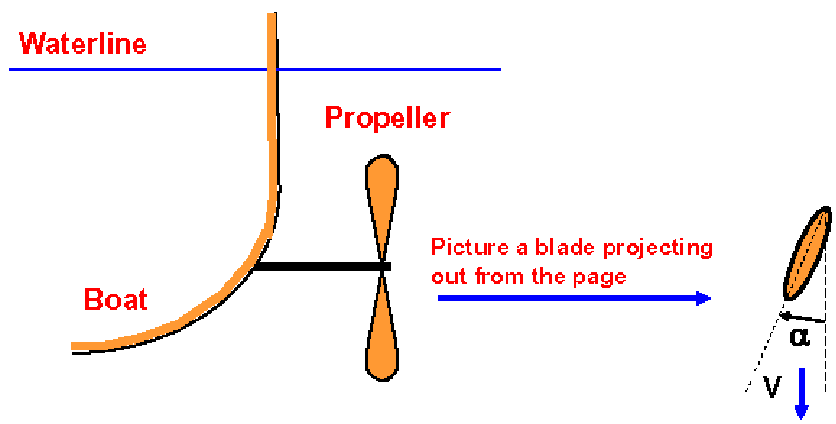
Solution to Problem 295B

(a) Picture a propeller blade moving through the water while the boat is at rest: The blade is moving through the water at



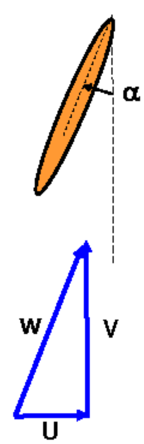
a velocity, V, and at an angle of attack, α, so it produces lift in the direction of forward propulsion of the boat. The four blades therefore produce a thrust equal to

$$\text{Thrust} = 4C_L sc\left(\frac{1}{2}\rho V^2\right)$$

where C_L is the lift coefficient of the blade at the angle of attack α, s is the span of the blade, and c is the chord of the blade. If we assume that $C_L \approx 2\pi\alpha$ then

$$\text{Thrust} = 4\pi\alpha sc\rho V^2$$

(b) Now consider what changes when the boat is in motion with a forward velocity, U. Then the fluid velocity relative to the blade has two components of velocity, V and U, as shown in the following sketch:



The angle of incidence with which the flow approaches the blade has now changed to $\alpha - (U/V)$ (assuming small angles so that $\tan^{-1}(U/V) \approx U/V$). Then the modified thrust is

$$\text{Thrust} = 4\pi(\alpha - (U/V))sc\rho V^2$$

At cruising speed this must equal the drag on the hull which we can estimate as

$$\text{Hull Drag} = C_D A\left(\frac{1}{2}\rho U^2\right)$$

where A is the hull area on which C_D is based. Equating the thrust and the hull drag

$$4\pi(\alpha - (U/V))scV^2 = C_D A\left(\frac{1}{2}U^2\right)$$

Rearranging the above equation

$$\frac{C_D A}{4\pi sc} = \alpha\frac{V^2}{U^2} - \frac{V}{U}$$

and solving for V/U yields the following expression for the cruising speed:

$$U = \frac{2\alpha V}{1 + \left[1 + \frac{C_D A\alpha}{\pi sc}\right]^{\frac{1}{2}}}$$