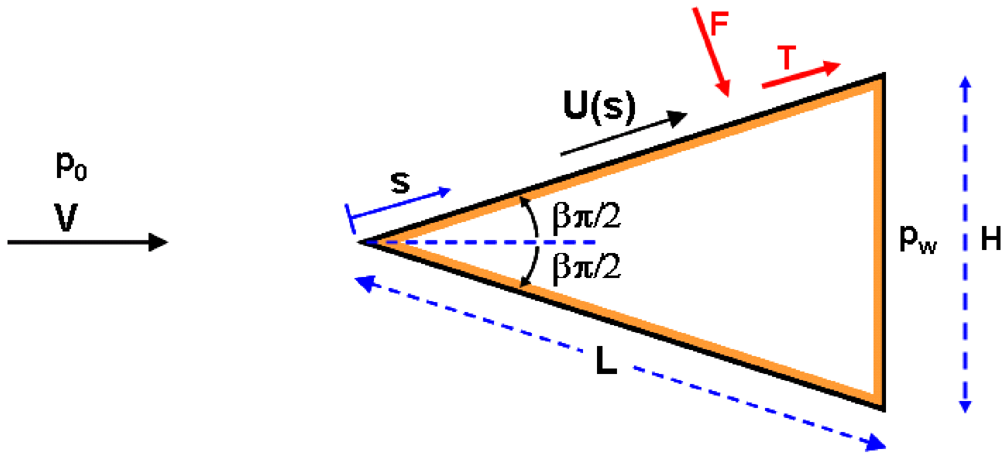
Solution to Problem 280B

The velocity outside of the boundary layer on the side of the wedge, $U(s)$ (where s is a coordinate measured along the side from the vertex of the wedge), is given by

$$U(s) = C\ s^m$$

From the potential flow solution past a wedge we know that $m = \beta/(2-\beta)$. Also we are given that the velocity at the rear corner, $U(L)$, is equal to V; therefore $C = V/L^m$.



As shown in the figure above, we will denote the pressure in the oncoming free stream by p_0, the uniform pressure in the wake by p_w, the force (per unit dimension normal to the sketch) on the side due to the pressure by F and the force (per unit dimension normal to the sketch) on the side due to the shear stress by T. It follows from applying Bernoulli's equation to the incompressible, irrotational flow outside of the boundary layer that the pressure, $p(s)$, on the side is given by

$$p_0 + \frac{1}{2}\rho V^2 = p(s) + \frac{1}{2}\rho U^2$$

where $U(s)$ is the velocity on the side outside of the boundary layer. Therefore

$$p(s) = p_0 + \frac{1}{2}\rho V^2 - \frac{\rho V^2}{2L^{2m}} s^{2m}$$

Integrating this from $s = 0$ to $s = L$ gives us the force, F, acting normal to the side (per unit dimension normal to the sketch):

$$F = p_0 L + \rho V^2 L \frac{m}{(2m+1)} = p_0 L + \rho V^2 L \frac{\beta}{(2+\beta)}$$

Moreover, the force acting on the back of the wedge in the direction upstream (per unit dimension normal to the sketch) is $p_w H = p_0 H$ and therefore the form drag, D_F, or force on the wedge due to the pressures acting on its faces, is

$$D_F = 2F sin\frac{\beta\pi}{2} - p_0 H = \frac{\beta}{(2+\beta)}\rho V^2 H$$

and the corresponding form drag coefficient is therefore

$$C_{DF} = \frac{D_F}{\frac{1}{2}\rho V^2 H} = \frac{2\beta}{(2+\beta)}$$

Now for the skin friction drag. The shear stress, τ_w, on the sides is given by

$$\frac{\tau_w}{\rho} = A(\beta)\frac{\nu^{\frac{1}{2}} U^{\frac{3}{2}}}{s^{\frac{1}{2}}}$$

and, substituting for $U(s)$, this becomes

$$\tau_w = \rho A(\beta)\nu^{\frac{1}{2}} C^{\frac{3}{2}} s^{\frac{(3m-1)}{2}}$$

Integrating this from $s = 0$ to $s = L$ gives us the force, T, acting tangential to the side (per unit dimension normal to the sketch):

$$T = \rho A(\beta)\nu^{\frac{1}{2}} V^{\frac{3}{2}} L^{\frac{1}{2}} \frac{2}{(3m+1)}$$

or

$$T = \frac{(2-\beta)}{(1+\beta)} A(\beta)\rho\nu^{\frac{1}{2}} V^{\frac{3}{2}} L^{\frac{1}{2}}$$

Consequently the drag, D_S, due to these shear forces

$$D_S = 2T cos\frac{\beta\pi}{2} = 2\frac{(2-\beta)}{(1+\beta)} A(\beta)\rho\nu^{\frac{1}{2}} V^{\frac{3}{2}} L^{\frac{1}{2}} cos\frac{\beta\pi}{2}$$

and the corresponding skin friction drag coefficient, C_{DS}, is therefore

$$C_{DS} = \frac{D_S}{\frac{1}{2}\rho V^2 H} = 2\frac{(2-\beta)}{(1+\beta)} A(\beta) \left(\frac{\nu}{VL}\right)^{\frac{1}{2}} cot\frac{\beta\pi}{2}$$

or

$$C_{DS} = 2\frac{(2-\beta)}{(1+\beta)} (0.332 + 0.87\beta)(Re_L)^{-\frac{1}{2}} cot\frac{\beta\pi}{2}$$

where $Re_L = VL/\nu$ is the Reynolds number of the flow.

It follows that for a 7.5^o half-angle wedge that $C_{DF} = 0.08$ and $C_{DS} = 10.87/(Re_L)^{\frac{1}{2}}$. Consequently for this angle wedge, the form drag and the skin friction drag will be equal at a Reynolds number, $Re_L = 1.85 \times 10^4$. For higher Reynolds numbers the form drag will dominate; for smaller Reynolds numbers the skin friction drag will dominate.