Solution to Problem 260B

To find the distance, x_{crit}, from the leading edge of the plate to the point where transition to turbulence begins, we note from the stability diagram that the critical Reynolds nnumber, $Re_{\delta^*,\text{crit}}$, is

$$Re_{\delta^*,\text{crit}} = \frac{U\delta^*_{\text{crit}}}{\nu} \approx 550$$

Using the Blasius laminar boundary layer solution we also know the expression for the displacement thickness as a function of x:

$$\frac{\delta^*}{x}\sqrt{Re_x} = 1.721$$

and so it follows that

$$\begin{aligned}
x_{\text{crit}} &= \left(\frac{\delta^*_{\text{crit}}}{1.721}\right)^2 \frac{U}{\nu} \\
&= \frac{\nu}{U}\left(\frac{Re_{\delta^*,\text{crit}}}{1.721}\right)^2 \\
&= \frac{10^{-6}}{2}\left(\frac{550}{1.721}\right)^2 = 0.0511 \text{ m}
\end{aligned}$$

To find the frequency, f, of the most unstable disturbance we also note from the stability diagram that the frequency which becomes unstable at the critical Reynolds number is

$$\frac{2\pi f\nu}{U^2} = 1.70 \times 10^{-4}$$

and therefore

$$f = \frac{1.70 \times 10^{-4}(2)^2}{2\pi(10^{-6})} = 108.2 \text{ Hz}$$