Solution to Problem 255B

First to estimate the distance, x_s, from the minimum pressure point to the laminar boundary layer separation point, we use Thwaites' method to determine the position of laminar boundary layer separation. Thwaites' method is based on the parameter, λ, defined as:

$$\lambda = \frac{\delta_m^2}{\nu}\frac{dU_\infty}{dx}$$

$$\delta_m^2 = (\delta_m)^2_{x=0} + \frac{0.45\nu}{U_\infty^6}\int_0^x U_\infty^5 dx$$

where δ_m is the momentum thickness of the boundary layer, $(\delta_m)_{x=0}$ is the momentum thickness at $x = 0$, U_∞ is the velocity just outside the boundary layer, ν is the kinematic viscosity and x is the streamwise surface coordinate.

Proceeding to calculate λ for this case:

$$\begin{aligned}
\delta_m^2 &= \frac{0.45\nu}{U_{max}^6}\left(1-cx^2\right)^{-6}\int_0^x U_{max}^5\left(1-cx^2\right)^5 dx \\
&= \frac{0.45\nu}{U_{max}}\left(1+6cx^2+\ldots\right)\int_0^x \left(1-5cx^2+\ldots\right)dx \\
&= \frac{0.45\nu}{U_{max}}\left(1+6cx^2+\ldots\right)\left(x-\frac{5}{3}cx^2+\ldots\right) \\
&\approx \frac{0.45\nu}{U_{max}}x
\end{aligned}$$

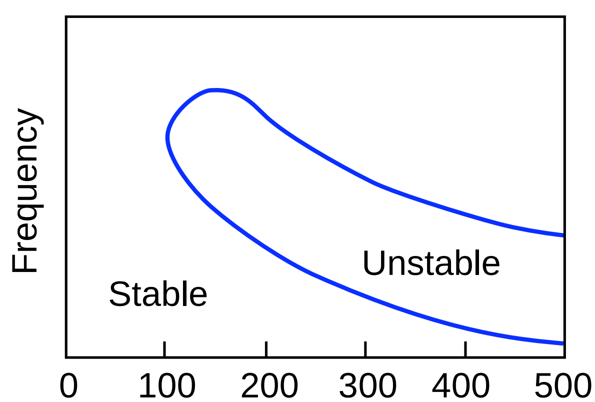
and therefore

$$\lambda = \left(\frac{0.45\nu}{U_{max}}x\right)\frac{1}{\nu}\left(-2U_{max}cx\right) = -0.9cx^2$$

The method supposes separation to occur at $\lambda = -0.09$ and therefore the distance, x_s, to the laminar separation point is:

$$x_s = \sqrt{\frac{1}{10c}}$$

Second we seek an expression for the distance, x_t, from the minimum pressure point to the point at which the boundary layer becomes unstable. The given stability diagram tells us that the boundary layer becomes unstable at:



$$\frac{\delta_m U_\infty}{\nu} \approx 100$$

Using the above momentum thickness calculated during Thwaites' method:

$$\left(\frac{0.45\nu}{U_{max}}x_t\right)^{1/2}\frac{U_{max}\left[1-cx^2\right.}{\nu}\approx\left(\frac{0.45\nu}{U_{max}}x_t\right)^{1/2}\frac{U_{max}\left[1\right]}{\nu}=100$$

and therefore the distance, x_t, from the minimum pressure point to the point at which the boundary layer becomes unstable is given by

$$x_t = 2.2\times 10^4\frac{\nu}{U_{max}}$$

Third, to find the desired critical Reynolds number, $Re = U_{max}d/\nu$, at which the boundary layer will become unstable just as it is about to separate we will equate the above expressions for x_s and x_t. In doing so note that since $d = c^{\frac{1}{2}}$ is given as the typical linear dimension of the body, we can rewrite x_s as:

$$x_s = \frac{d}{\sqrt{10}}$$

and therefore the boundary layer will become unstable just as it is about to separate when

$$\frac{d}{\sqrt{10}} = 2.22\times 10^4\frac{\nu}{U_{max}}$$

so that the critical Reynolds Number is

$$Re = \frac{U_{max}d}{\nu} = 7.027\times 10^4$$