Solution to Problem 250F

The Karman Momentum Integral Equation is :

$$\frac{\tau_w}{\rho} = \frac{d}{dx}\left(U^2\delta_M\right) + \delta_D U \frac{dU}{dx}$$

where α, β and γ are the usual profile parameters, τ_w is the wall shear stress, U is the velocity exterior to the boundary layer, δ is the boundary layer thickness, ν is the kinematic viscosity of the fluid and x is the streamwise distance along the wall surface.

According to the definitions of $\beta = \frac{d(u/U)}{d(y/\delta)}$ and $\tau_w = \mu\frac{du}{dy}$, the left hand side of the Karman Momentum Integral Equation (KMIE) can be expressed as:

$$\frac{\tau_w}{\rho} = \frac{\nu U \beta}{\delta}$$

The definitions of α and γ give:

$$\delta_M = \alpha\delta$$

$$\delta_D = \gamma\delta$$

Substitute into the KMIE:

$$\begin{aligned}
\frac{\nu U \beta}{\delta} &= \frac{d}{dx}\left(U^2\alpha\delta\right) + \gamma\delta U\frac{dU}{dx} \\
&= \alpha U^2\frac{dU}{dx} + 2\alpha\delta U\frac{d\delta}{dx} + \gamma\delta U\frac{dU}{dx} \\
\frac{\nu\beta}{\alpha\delta} &= U\frac{d\delta}{dx} + \left(2 + \frac{\gamma}{\alpha}\right)\delta\frac{dU}{dx}
\end{aligned}$$

Substituting $U = Ax^{\frac{1}{2}}$ and $\delta = Cx^m$ yields:

$$\frac{\nu\beta}{\alpha C x^m} = mACx^{m-\frac{1}{2}} + \left(1 + \frac{\gamma}{2\alpha}\right)ACx^{m-\frac{1}{2}}$$

By matching powers of x,

$$m = \frac{1}{4}$$

and

$$C = \left[\frac{4\nu\beta}{\alpha A(5 + \frac{2\gamma}{\alpha})}\right]^{\frac{1}{2}}$$