Solution to Problem 250E

The Karman Momentum Integral Equation is :

$$\frac{\tau_w}{\rho} = \frac{d}{dx}\left(U^2\delta_M\right) + \delta_D U\frac{dU}{dx}$$

where α, β and γ are the usual profile parameters, τ_w is the wall shear stress, U is the velocity exterior to the boundary layer, δ is the boundary layer thickness, ν and ρ are the kinematic viscosity and density of the fluid and x is the streamwise distance along the wall surface.

It follows that if $U = Ax$ the boundary layer thickness, δ, will be given by

$$\frac{\nu\beta Ax}{\delta} = \frac{d}{dx}\left(\alpha\delta A^2x^2\right) + \delta\gamma A^2x$$

or

$$\frac{\nu\beta}{A\delta} = \alpha x\frac{d\delta}{dx} + \delta(2\alpha+\gamma)$$

for which the only reasonable solution is

$$\delta = \text{constant} = \left[\frac{\nu\beta}{A(2\alpha+\gamma)}\right]^{\frac{1}{2}}$$