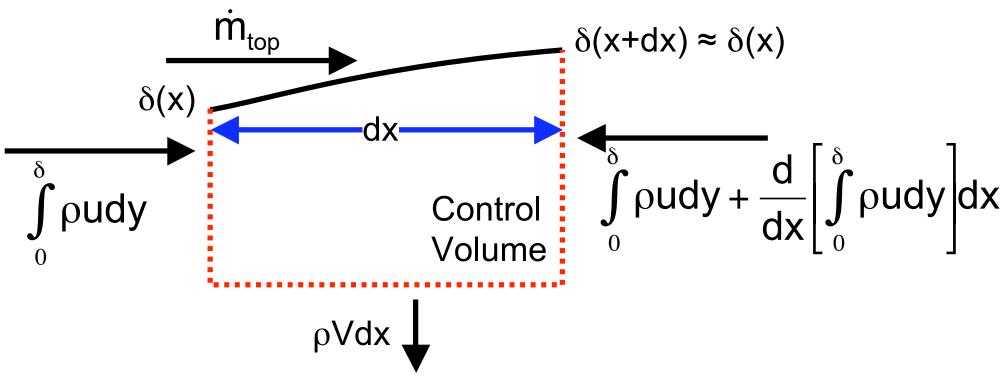
Problem 250B



We define a control volume between x and $x + dx$, bounded by the surface of the plate and extending to the edge of the boundary layer. Then continuity requires that the mass flow rate into the top of the control volume, M, is:

$$\dot{m}_{top} \;=\; \frac{d}{dx}\left[\int_0^\delta \rho u dy\right] dx + \rho V dx \tag{1}$$

Now, preparing to apply the momentum theorem in the x direction, the net flux of x-momentum out of the control volume in the x-direction, $\dot{M}_x$, is

$$\dot{M}_x \;=\; \frac{d}{dx}\left\{\int_0^\delta \rho u^2 dy\right\} dx - \; \dot{m}_{top} U \tag{2}$$

(Note that there is no x-direction momentum flux through the plate surface). After substituting for $\dot{m}_{top}$ this becomes

$$\dot{M}_x \;= \frac{d}{dx}\left\{\int_0^\delta \rho u^2 dy\right\} dx - \left[\frac{d}{dx}\left\{\int_0^\delta \rho u dy\right\} dx + \rho V dx\right] U$$

$$\dot{M}_x \;=\; \rho U^2 \left[-\frac{d}{dx}\left\{\int_0^\delta \frac{u}{U}\left(1 - \frac{u}{U}\right) dy\right\} - \frac{V}{U}\right] \; dx$$

$$\dot{M}_x \;=\; \rho U^2 \left[-\alpha \frac{d\delta}{dx} - \frac{V}{U}\right] dx \tag{3}$$

Applying the momentum theorem we find that the force on the control volume in the x-direction, F_x, is:

$$\dot{M}_x \;=\; F_x \;=\; -\tau_w dx - \frac{dp}{dx}\delta(x) dx \tag{4}$$

and since there is no pressure gradient in the exterior flow and therefore no force due to a pressure difference it follows that

$$\tau_w \;=\; \rho U^2 \left[\alpha \frac{d\delta}{dx} + \frac{V}{U}\right] \tag{5}$$

The skin friction coefficient C_f is then given by:

$$C_f \;=\; \frac{\tau_w}{\frac{1}{2}\rho U^2} \;=\; 2\left[\alpha\frac{d\delta}{dx}+\frac{V}{U}\right] \tag{6}$$

Thus the primary effect of the suction is to increase the skin friction through the $+V/U$ terms in the above equations though this effect is somewhat offset by a reduction in $d\delta/dx$, an effect which can be shown to be secondary. Consequently the suction reduces the potential for flow separation since laminar boundary layer separation occurs when the friction decreases to zero. This effect is sometimes used on a portion of the surface of an airfoil in order to prevent or, at least, delay flow separation and thus improve the performance of the airfoil.