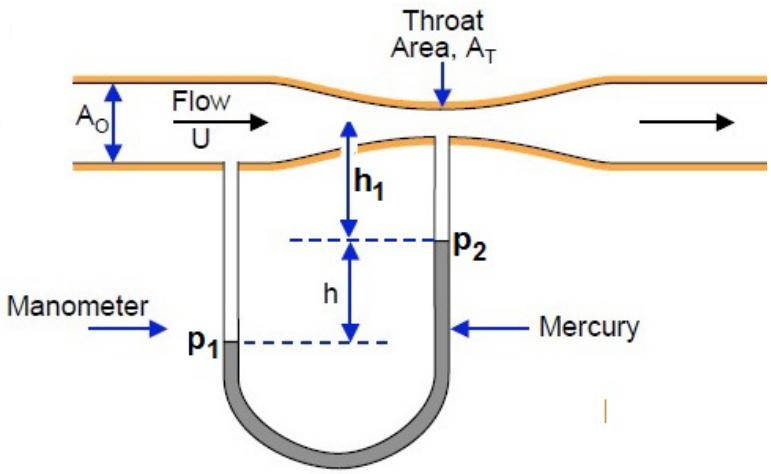
Solution to Problem 220H:

The following device, known as a Venturi meter, is used to measure the flow rate of water in a pipe of cross-sectional area A_0. A convergent/divergent nozzle with a throat area A_T is installed in the pipe: and



pressure taps are located upstream of the nozzle and at the throat; these are connected to a water/mercury manometer as shown. When the water is flowing through the device, the manometer levels differ by an elevation, h.

Continuity requires that

$$U\ A_0\ =\ U_T\ A_T \quad \text{so that} \quad U_T\ =\ U\frac{A_0}{A_T} \tag{1}$$

and Bernoulli's equation requires that

$$p_T - p_0\ =\ \frac{\rho_W}{2}U^2\left[1-\frac{A_0^2}{A_T^2}\right] \tag{2}$$

where p_T and p_0 are the pressures in the throat and far upstream. Therefore

$$U\ =\ \left[\frac{2A_T^2(p_0-p_T)}{\rho_W(A_0^2-A_T^2)}\right]^{1/2} \tag{3}$$

The manometer will read

$$p_0 - p_T\ =\ (\rho_M-\rho_W)gh \tag{4}$$

and therefore the flow rate, Q, can be determined using

$$Q\ =\ UA_0\ =\ \left[\frac{2ghA_T^2A_0^2(\rho_M-\rho_W)}{\rho_W(A_0^2-A_T^2)}\right]^{1/2} \tag{5}$$