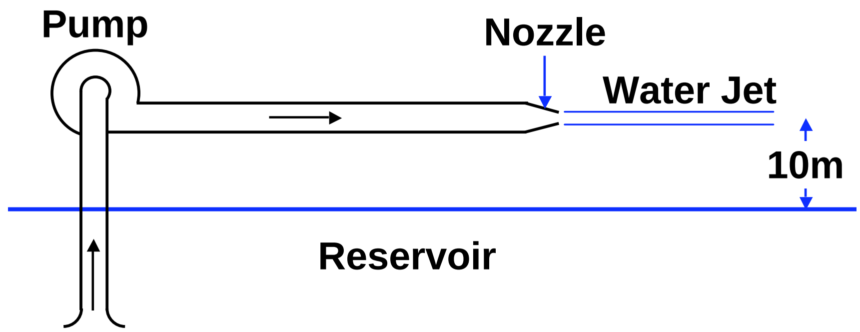
Solution to Problem 220J

A fire nozzle is to be used at an elevation of 10 m above the level of a reservoir. The velocity of the jet is to be 20 m/s and the flow is provided by a pump:



A modified Bernoulli equation applied between a point on the external free surface outside the submerged pipe (point 1) and the jet after it emerges from the nozzle (point 2) yields

$$\frac{p_A}{\rho g} + H - k_1 \frac{u_1^2}{2g} - k_2 \frac{u_1^2}{2g} = \frac{p_A}{\rho g} + \frac{u_2^2}{2g} + y_2$$

where the elevation of the external free surface is taken to be $y = 0$. In the above p_A is the atmospheric pressure, the pressure (and total pressure) at the point 1 is p_A and it is assumed that the pressure in the emerging jet is equal to p_A. Also u_1 is the fluid velocity in the pipes leading to and from the pump, u_2 is the fluid velocity in the jet, y_2 is the elevation of the emerging jet, ρ is the density of the liquid, H is the total head rise across the pump, and k_1 and k_2 are the loss coefficients for the pipes leading to and from the pump.

It also follows from continuity that

$$A_1 u_1 = A_2 u_2$$

where A_1 and A_2 are the cross-sectional areas of the pipes and the jet respectively. Since $u_2 = 20\ m/s$ it follows that $u_1 = 2\ m/s$ and that

$$H = \frac{u_2^2}{2g} \left[1 + \left(\frac{A_2^2}{A_1^2} \right) (k_1 + k_2) \right] + y_2$$

Substituting $u_2 = 20\ m/s$, $y_2 = 10\ m$, $A_2/A_1 = 0.1$, $g = 9.8\ m/s^2$, $k_1 + k_2 = 7$ it follows that

$$H = \frac{400}{19.6} \left[1 + (0.01)(7) \right] + 10 = 31.8m$$

and this is the total head the pump must provide. Moreover since the mass flow rate is $1000 \times 2 \times 0.007 = 14\ kg/s$ it follows that the power that must be supplied to the fluid in the pump is $14 \times 31.8 \times 9.8 = 4363\ kg\ m^2/s^3$. Therefore the power that must be supplied to the pump is $4363/0.75 = 5817\ kg\ m^2/s^3$.

If we take a control volume surrounding the pump, the discharge pipe and the nozzle but cutting through the emerging jet and the pipe leading to the pump, the horizontal momentum flux emerging from this control volume (in the direction of the jet) is $\rho A_2 u_2^2$ which equals 280 $kg\ m/s^2$. By the momentum thereom, since the pressure on all horizontal sides of the control volume is atmospheric and therefore the net pressure force in the horizontal direction is zero, the force that must be applied to the vehicle to hold it in place is also 280 $kg\ m/s^2$ in the direction of the jet.