Solution to Problem 220C

Conservation of mass requires that the exhaust jet velocity, $U_J = UA_1/A_J$ or $Q = UA_1 = U_J A_J$. Therefore, the total head rise across the propeller is given by

$$\begin{aligned} \frac{U_J^2}{2g} - \frac{U_1^2}{2g} &= \Delta H \\ &= \frac{K_1 - K_2 Q}{g} \end{aligned}$$

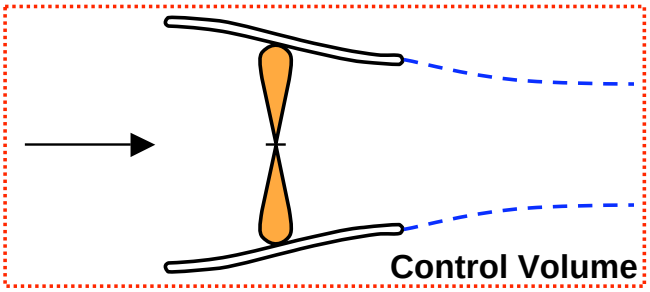
which, substituting for U_J and U, yields

$$\frac{Q^2}{2gA_J^2} - \frac{Q^2}{2gA_1^2} = \frac{K_1 - K_2 Q}{g}$$

and therefore

$$A_J = \frac{UA_1}{[U^2 + 2(K_1 - K_2 UA_1)]^{\frac{1}{2}}}$$

Now take a large control volume around the jet. The thrust, T, produced by the ducted propeller in the x direction is equal



to the net flux of x-momentum out of the control volume so that

$$T = \rho A_J U_J^2 - \rho A_1 U^2$$

because, provided the control volume is much larger than the structure, there are no pressure forces. Substituting for A_J,

$$T = \rho A_1 U^2 \left[\left\{ 1 + \frac{1(K_1 - K_2 UA_1)}{U^2} \right\}^{\frac{1}{2}} - 1 \right]$$