Solution to Problem 205C

The weight of the vehicle, W, must be balanced by the force exerted by the difference in pressure inside and outside the vehicle,

$$W = (p_c - p_a)\, A_p = (p_c - p_a)\, \pi R^2$$

where A_p is the projected area of the vehicle, which gives

$$p_c - p_a = \frac{W}{\pi R^2} = A - BQ$$

from the information given in the problem statement. The modified Bernoulli equation for the flow through the gap, h, gives

$$p_c - p_a = \frac{1}{2}\rho u^2 \frac{1}{k}$$

where the coefficient $k = 1$ and u is the velocity through the gap.

By the definition of the flow rate, it follows that

$$Q = A_g u = 2\pi R h u$$

where A_g is the area of the gap. Rearranging and using the expression for u, it follows that

$$h = \frac{Q}{2\pi R u} = \frac{Q}{2\pi R}\left[\frac{\rho}{2(p_c - p_a)}\right]^{\frac{1}{2}}$$

Substituting for Q and $p_c - p_a$

$$h = \left(\frac{1}{2\pi R}\right)\left(\frac{1}{B}\right)\left(A - \frac{W}{\pi R^2}\right)\left(\frac{\rho \pi R^2}{2W}\right)^{\frac{1}{2}}$$

which simplifies to

$$h = \frac{1}{B}\left(A - \frac{W}{\pi R^2}\right)\left(\frac{\rho}{8\pi W}\right)^{\frac{1}{2}}$$