Solution to Problem 204A:

Since the inlet swirl is zero the torque, T, applied to the fluid flow (density ρ) by the fan is given by

$$T \quad \propto \quad m r_2 v_{\theta_2} \tag{1}$$

where m is the mass flow rate through the fan, r_2 is the radius of the discharge flow (we assume for simplicity that the exit flow is all at the same radius r_2) and v_{θ_2} is the discharge swirl velocity. Moreover the basic angular momentum equation applied to the fan yields

$$T\Omega \quad = \quad mgH \tag{2}$$

and therefore the total head rise, H, across the fan is given by

$$H \quad = \quad \frac{r_2 v_{\theta_2} \Omega}{g} \tag{3}$$

Therefore $v_{\theta_2} = gH/\Omega r_2$.

Now the kinetic energy which is assumed lost in the discharge swirl is given by $\frac{1}{2} m v_{\theta_2}^2$ while the total energy in the discharge flow is given by mgH so the fraction of the energy dissipated in the discharge swirl is given by

$$\frac{\text{Lost discharge swirl energy}}{\text{Total discharge energy}} = \frac{0.5 m v_{\theta_2}^2}{mgH} = \frac{\psi}{2} \tag{4}$$