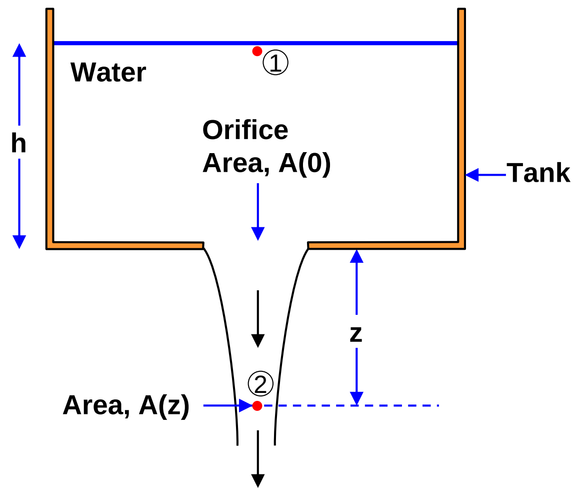
Solution to Problem 201A

Because the height of the tank is being kept constant, the flow can be assumed to be steady. Then labelling the surface of



the water in the tank as point 1 and a point a distance z below the orifice in the bottom of the tank as point 2, the steady form of Bernoulli's equation yields:

$$\frac{p_1}{\rho} + \frac{1}{2}u_1^2 + gz_1 = \frac{p_2}{\rho} + \frac{1}{2}u_2^2 + gz_2$$

where since $A(0) \ll A_1$, the velocity at point 1 can be neglected.

Denoting the atmospheric pressure at the surface of the water in the tank by p_A, then $p_1 = p_A$. Assuming that the velocity is uniform across the jet, it can be concluded from Bernoulli's equation that the pressure is also constant across the jet and must therefore be equal to p_A, the pressure on the sides of the jet. Then:

$$\frac{p_A}{\rho} + gh = \frac{p_A}{\rho} + \frac{1}{2}u_2^2 - gz.$$

and solving for u_2 yields

$$u_2(z) = \sqrt{2g(z+h)}$$

Also from conservation of mass, it follows that

$$A(0)u_2(0) = A(z)u_2(z)$$

and substituting for $u_2(z)$ and $u_2(0)$ from the above relation this yields

$$A(z) = \sqrt{\frac{h}{z+h}}A(0)$$