## Solution to Problem 150M

This problem was discussed and presented in class. The solution obtained to the general problem gave the fluid velocity parallel to the plate, $u$, as a function of the distance from the plate, $y$, and the time, $t$, as

$$u(y,t) = U\left[1 - erf\left(\frac{y}{(4\nu t)^{\frac{1}{2}}}\right)\right]$$

where $U$ is the velocity of the plate, $\nu$ is the kinematic viscosity of the fluid and $erf()$ is the error function. In addition the vorticity, $\omega(y,t)$, was found to be

$$\omega(y,t) = \frac{U}{(\pi\nu t)^{\frac{1}{2}}} exp\left(-\frac{y^2}{(4\nu t)}\right)$$

Therefore, using the above expression for $u(y,t)$, we find that the time, $t*$, at which the velocity becomes 0.5 $m/s$ at a point that is $y = 0.01m$ from the plate is

$$0.5\ m/s = 1.0\ m/s\ \left[1 - erf\left(\frac{0.01}{(4 \times 10^{-6} \times t*)^{\frac{1}{2}}}\right)\right]$$

which, using the given fact that $erf(z) = 0.5$ when $z = 0.475$, yields

$$t* \ = 110.8\ s$$

Also the above expression for the vorticity permits us to evaluate both the vorticity at the plate, $\omega(0,t)$, and the vorticity at $y = 0.01\ m$, namely $\omega(0.01,t)$:

$$\omega(0,t) = \frac{1.0}{(\pi \times 10^{-6} \times t)^{\frac{1}{2}}}$$

$$\omega(0.01,t) = \frac{1.0}{(\pi \times 10^{-6} \times t)^{\frac{1}{2}}} exp\left(-\frac{0.01^2}{(4 \times 10^{-6} \times t)}\right)$$

and therefore at $t = 110.8$ the ratio of $\omega(0.01,t)$ to $\omega(0,t)$ becomes:

$$\frac{\omega(0.01,t)}{\omega(0,t)} = exp\left(-\frac{0.01^2}{(4 \times 10^{-6} \times 110.8)}\right) = 0.798$$