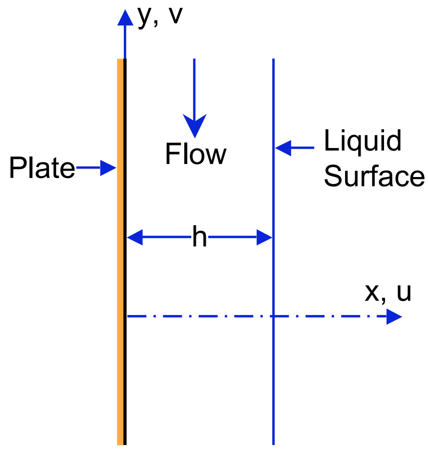
Solution to Problem 150H:

A constant and uniform layer of Newtonian, viscous, incompressible liquid (dynamic viscosity, μ, and density, ρ) flows down a vertical flat plate:



Consider a control volume spanning the whole layer, with unit height in the y direction and unit depth normal to the diagram. The weight of the liquid in this control volume is $\rho g h$ and this must be balanced by the upward shear force at the wall (since the flow is steady and since the pressures acting on the top and bottom of this control volume are the same). Since the velocity in the y direction is

$$v(x) \; = \; -Cx(2h - x)$$

the shear stress at the wall must be

$$-\mu \left(\frac{dv}{dx} \right)_{x=0} \; = \; \mu C \left(2h - 2x \right)_{x=0} \; = \; 2\mu C h$$

and since this must be equal $\rho g h$ it follows that

$$C \; = \; \frac{\rho g}{2\mu}$$