Solution to Problem 122A

The velocity potential for flow around a sphere of radius R is created by superposition of uniform flow and a 3-D doublet:

$$\begin{aligned} \phi &= \underbrace{Ux}_{\text{Uniform stream}} + \underbrace{A\frac{\cos\theta}{r^2}}_{\text{3-D Doublet}} \\ &= \left(Ur + \frac{A}{r^2}\right)\cos\theta \end{aligned}$$

Thus the radial velocity, u_r, is given by:

$$u_r = \frac{\partial \phi}{\partial r} = \left(U - \frac{2A}{r^3}\right)\cos\theta$$

The zero normal velocity condition ($u_r = 0$) must hold at the surface of the sphere ($r = R$) so:

$$u_r|_{r=R} = \left(U - \frac{2A}{R^3}\right)\cos\theta = 0$$

Therefore the constant A must be:

$$A = \frac{UR^3}{2}$$

Thus the velocity potential for flow over a sphere is given by:

$$\phi = Ur\left[1 + \frac{1}{2}\left(\frac{R}{r}\right)^3\right]\cos\theta$$

The only component of the velocity on the surface of the sphere will be that in the tangential direction, u_θ. Therefore, the maximum velocity on the surface of the sphere will occur where u_θ is a maximum and, since,

$$u_\theta = \frac{1}{r}\frac{\partial \phi}{\partial \theta} = -U\left[1 + \frac{1}{2}\left(\frac{R}{r}\right)^3\right]\sin\theta$$

the velocity on the surface of the sphere $r = R$ is given by

$$u_\theta|_{r=R} = -\frac{3}{2}U\sin\theta$$

This velocity will be a maximum when

$$|\sin\theta| = 1$$

or where

$$\theta = \frac{\pi}{2}, \frac{3\pi}{2}$$

Thus, the maximum velocity is

$$\max\left(|u_\theta|_{r=R}\right) = \frac{3}{2}U$$