Solution to Problem 121A

As derived in class, the pressure distribution on the surface of a cylinder in potential flow is given by $p(\alpha)$ where:

$$p(\alpha) = p_\infty + \frac{1}{2}\rho U^2 (1 - 4\sin^2\alpha)$$

where α is the angle measured from the front or rear stagnation point. Note that as $p(\alpha)$ is symmetric about $\alpha = \pi/2$ and it does not matter whether the angle is measured from the front or rear stagnation point. The above expression describes therefore the pressure acting on the outside of the arctic hut where p_∞ is the pressure far away, ρ is the (incompressible) fluid density and U is the velocity of the cross-wind. The pressure inside of the hut is p_I where

$$p_I = p_\infty + \frac{1}{2}\rho U^2 (1 - 4\sin^2\theta)$$

where θ is the angle of the vent location. The difference in pressure between the inside and outside of the hut is then given by

$$p - p_I = 2\rho U^2 \left(\sin^2\theta - \sin^2\alpha\right)$$

and the components of force on the hut are then obtained by integrating this difference over the arc and multiplying by the appropriate angle. Therefore

$$\begin{aligned} F_y &= \int_0^\pi 2\rho R U^2 \left(\sin^2\theta - \sin^2\alpha\right)\sin(\alpha) d\alpha \\ &= 2\rho R U^2 \left(2 sin^2\theta - \frac{4}{3}\right) \end{aligned}$$

It follows that the lift on the hut will be zero when $\sin^2\theta = 2/3$ or when $\theta = 54.7^\circ$ or 125.3°