Solution to Problem 115G

The velocity, u, in the x direction for this planar incompressible flow is

$$u = U\left\{\frac{2y}{ax} - \frac{y^2}{a^2x^2}\right\}$$

where a is a constant. Since $u = \partial\psi/\partial y$, where ψ is the streamfunction, it follows that

$$\frac{\partial\psi}{\partial y} = U\left\{\frac{2y}{ax} - \frac{y^2}{a^2x^2}\right\}$$

and this can be integrated with respect to y to yield

$$\psi = U\left\{\frac{y^2}{ax} - \frac{y^3}{3a^2x^2}\right\} + C(x)$$

where $C(x)$ is the integration constant, an unknown function of x alone. Then, differentiating with respect to x we obtain the velocity, v, in the y direction:

$$v = -\frac{\partial\psi}{\partial x} = U\left\{\frac{y^2}{ax^2} - \frac{2y^3}{3a^2x^3}\right\} + \frac{dC}{dx}$$

where dC/dx will also just be a function of x.

But we also know that, at the wall $y = 0$, we must have zero velocity, v, normal to the wall and therefore, from the last equation, dC/dx must be zero at the wall, $y = 0$. But since dC/dx is only a function of x dC/dx must therefore be zero everywhere and hence

$$v = -\frac{\partial\psi}{\partial x} = U\left\{\frac{y^2}{ax^2} - \frac{2y^3}{3a^2x^3}\right\}$$