Solution to Problem 115A

Given $u = A(y - ax)$ and that $u = \partial\psi/\partial y$ it follows that

$$\frac{\partial \psi}{\partial y} = A(y - ax)$$

and therefore by integration

$$\psi = \frac{Ay^2}{2} - Aaxy + c(x)$$

Note $c(x)$ is an arbitrary constant in ψ which we can set to zero. It follows that

$$v = -\frac{\partial \psi}{\partial x} = Aay$$

In summary, the streamfunction and velocities are

$$\begin{aligned} \psi &= \frac{Ay^2}{2} - Aaxy \\ u &= A(y - ax) \\ v &= Aay \end{aligned}$$

Now consider the conditions at the sloping wall

$$\begin{aligned} \frac{\Delta y}{\Delta x} = b = \frac{v}{u} &= \frac{Aay}{A(y - ax)} \quad \text{where} \quad y = bx \\ b &= \frac{Aabx}{A(bx - ax)} \\ b &= \frac{ab}{(b - a)} \\ \therefore\ b &= 2a \end{aligned}$$