Solution to Problem 112B

Since chemical constituents are carried along with the fluid

$$\frac{\mathrm{D}c}{\mathrm{D}t} = \frac{\partial c}{\partial t} + u\frac{\partial c}{\partial x} = \alpha$$

But the flow is steady and therefore $\frac{\partial c}{\partial t} = 0$ and $c(x)$ is only a function of x. Therefore,

$$\frac{\partial c}{\partial x} = \frac{\alpha}{u} = \frac{\alpha}{u_0}\left(\frac{x_0}{x}\right)^2$$

Integrating,

$$c = -\frac{\alpha x_0^2}{u_0 x} \quad + \quad \text{constant}$$

But $c = c_0$ at $x = x_0$. Therefore

$$c - c_0 = \frac{\alpha x_0^2}{u_0}\left[\frac{1}{x_0} - \frac{1}{x}\right]$$