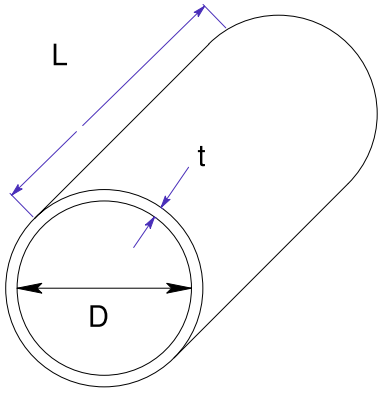
## Solution to Problem 109A




It is assumed that the flow is laminar and the fluid is Newtonian, such that the fluid displays a linear dependence of the shear stress on the shear rate. The experiment is conducted with fluid in between two concentric cylinders with one cylinder fixed. Since the radius, $D/2$, is much larger than the gap width, $t << D/2$, the flow is essentially Couette flow.
In the assignment, the parameters are as given in table 1.

| Parameter | Value | Units |
|---|---|---|
| diameter D | 0.02 | m |
| length L | 0.2 | m |
| thickness t | $1 \times 10^{-4}$ | m |
| dynamic viscosity $\mu$ | 0.1 | kg/ms |
| rotational speed n | 6000 | rpm |

Table 1: Given parameters.

The torque $T$ is defined as the force $F$ multiplied by the radius $D/2$:

$$T = F\frac{D}{2} = \sigma A\frac{D}{2}$$

where the shear stress *sigma* acts in the circumferential direction for Couette flow and can be written as:

$$\sigma = \mu\frac{dU}{dy} = \mu\frac{U}{t}$$

given the linear velocity profile. The velocity $U$ in this case can be calculated from:

$$U = n\frac{\pi D}{60} = 2\pi \ m/s \approx 6.283 \ m/s$$

Thus, the torque is given by:

$$\begin{aligned} T &= \sigma A\frac{D}{2} \\ &= \left(\mu\frac{U}{t}\right)(\pi DL)\left(\frac{D}{2}\right) \\ &= \mu\frac{n\pi^2 D^3 L}{120t} \approx 0.790\frac{kg\ m^2}{s^2} \end{aligned}$$

The power $P$ is defined as the torque $T$ multiplied by the angular velocity $\omega$:

$$\begin{aligned} P &= T\omega \\ &= T\frac{U}{D/2} \\ &= \mu\ \frac{n^2\pi^3 D^3 L}{3600t} \approx 496\frac{kg\ m^2}{s^3} \end{aligned}$$