Solution to Problem 108F

Three vertical forces act on the cube:

A. The weight of the cube, $\rho_s g L^3$

B. The vertical component of the surface tension force acting along the contact line on the sides of the cube, $4LS\cos\alpha$

C. The combination of the atmospheric pressure on the top of the cube and the water pressure on the bottom of the cube. The pressure on the bottom is atmospheric pressure plus $\rho_L g(h+B)$ where B is the distance from the bottom corner up the side to the contact line. Thus the upward force will be greatest when B is greatest and this condition will support the heaviest cube. Thus the heaviest cube will be supported when $B = L$. Under this condition the upward force resulting from the pressures on the top and bottom is $\rho_L g(h+B)L^2$

The balance of these three forces yields

$$\rho_L g(h+L)L^2 + 4LS\cos\alpha = \rho_S g L^3$$

Substituting for h and α yields

$$\rho_S/\rho_L = 1 + (S/\rho_L g L^2)^{\frac{1}{2}} + 4(S/\rho_L g L^2)/2^{\frac{1}{2}}$$

which for $S/\rho_L g L^2 = 0.1$ yields $\rho_S/\rho_L = 1.6$