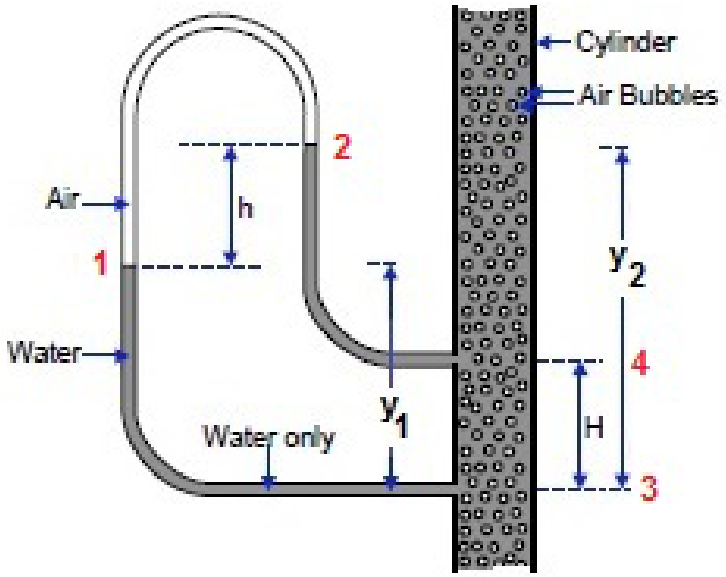
Solution to Problem 102D:



By definition the density is the ratio of the mass, m, to the volume, V, and in this two-phase flow the mass, m, is given by the sum of the mass of vapor and the mass of liquid:

$$m \quad = \quad \alpha\rho_A V + (1-\alpha)\rho_L V \tag{1}$$

where the air density is denoted by ρ_A and the water density by ρ_L. Therefore the effective mixture density, ρ, is given by

$$\rho \quad = \quad \alpha\rho_A + (1-\alpha)\rho_L \quad \approx \quad (1-\alpha)\rho_L \tag{2}$$

Tracing the pressure, p, around the manometer (denoting the :

$$p_1 \quad = \quad p_2 + \rho_A g h \quad ; \quad p_3 \quad = \quad p_1 + \rho_L g y_1 \tag{3}$$

$$p_4 \quad = \quad p_2 + \rho_L g(y_2 - H) \quad ; \quad p_3 \quad = \quad p_4 + \rho g H \tag{4}$$

where $y_2 = h + y_1$, $\rho \approx \rho_L(1-\alpha)$ and $\alpha = 1 - \rho/\rho_L$. By elimination

$$\alpha \quad = \quad h/H \tag{5}$$