Solution to Problem 101B

Given that up to 40 km the atmosphere of Venus behaves adiabatically, it follows that

$$p = C\rho^{\gamma} \tag{1}$$

where C is a constant. Now the pressure, p, as a function of the altitude, y, is given by

$$\frac{dp}{dy} = -\rho g.$$

and substituting we obtain

$$\frac{dp}{dy} = -\left(\frac{p}{C}\right)^{1/\gamma} g = -p^{1/\gamma}\left(\frac{1}{C}\right)^{1/\gamma} g.$$

Now separate the variables p and y and integrate (introducing the dummy variables ξ for p and η for y to avoid confusion):

$$\frac{d\xi}{\xi^{1/\gamma}} = -g\left(\frac{1}{C}\right)^{1/\gamma} d\eta$$

and using the constants p_s and ρ_s:

$$\int_{p_s}^{p} \xi^{-1/\gamma} d\xi = \int_0^y -g\left(\frac{1}{C}\right)^{1/\gamma} d\eta$$

$$\frac{1}{1-1/\gamma}\left(p^{1-1/\gamma} - p_s^{1-1/\gamma}\right) = -g\left(\frac{1}{C}\right)^{1/\gamma} y$$

$$p = \left[\left(-1+\frac{1}{\gamma}\right) g \left(\frac{1}{C}\right)^{1/\gamma} y + p_s^{1-1/\gamma}\right]^{\frac{1}{1-1/\gamma}}$$

The result for p is thus

$$p = \left[\left(\frac{1-\gamma}{\gamma}\right) g \left(\frac{\rho_s^{\gamma}}{p_s}\right)^{1/\gamma} y + p_s^{1-1/\gamma}\right]^{\frac{1}{1-1/\gamma}}$$

Part 2.

Plugging the values given in the problem statement into the expression for p:

$$p = \left[\left(\frac{1-1.2}{1.2}\right) 8.7 \left(\frac{63^{1.2}}{9.26\times10^6}\right)^{1/1.2} 30,000 + \left(9.26\times10^6\right)^{\left(1-\frac{1}{1.2}\right)}\right]^{\frac{1}{1-\frac{1}{1.2}}}$$

The pressure at an altitude of 30 km is thus

$$p = 1.13\times10^6 \text{ Pa}$$