Problem 420A

A bluff body with a frontal projected area of A_B is placed in the liquid flow in a pipe or tunnel of cross-sectional area, $A_T = \alpha A_B$. The liquid velocity far upstream of the body is adjusted to a constant value, U. The pressure far upstream of the body, p_∞, is then reduced until the wake behind the body becomes filled with vapor:

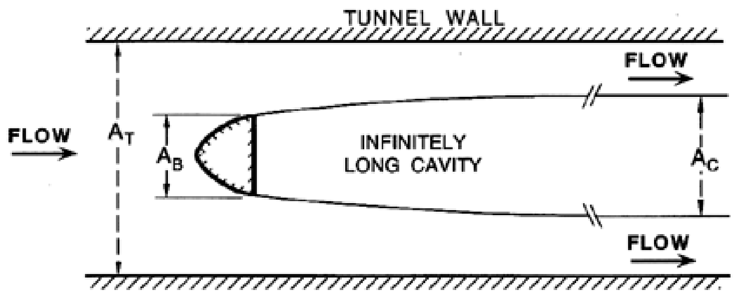

Figure 1: Body with infinitely long cavity under choked flow conditions.

As p_∞ is further reduced this fully-developed cavity increases in size. At a particular value of the cavitation number, $\sigma = 2(p_\infty - p_V)/\rho U^2$ (where p_V is the vapor pressure and ρ is the liquid density) this cavity tends to become infinitely long and the cross-sectional area of the cavity far downstream of the body tends to $A_C = \beta A_B$. Find the particular cavitation number, say σ_C, that corresponds to this asymptotic condition in terms of α and β assuming:

1. the liquid is incompressible and inviscid and the flow is irrotational
2. surface tension is neglected and gravity effects can be ignored
3. the amount of liquid vaporized to fill the cavity is negligible

Also: If, instead of fixing U, p_∞ and p_V are fixed and the asymptotic condition is approached by increasing the velocity, U. What is the maximum or choked tunnel velocity, U, that can be achieved in terms of p_∞, p_V, α and β?

Also: Find σ_C when condition (3) does not hold and the ratio of the vapor density to the liquid density is denoted by γ. Assume that far downstream of the body the liquid and vapor velocities are equal.