Problem 401A

[1] Consider a spherical bubble containing vapor and an insoluble gas in an incompressible, inviscid liquid (density ρ_L) whose pressure far from the bubble is denoted by p_∞. The surface tension at the bubble surface is denoted by S. The bubble pressure, p_B, is the sum of the vapor pressure, p_V, of the vapor within the bubble (p_V is a known constant) and the partial pressure of the insoluble gas, p_G. The mass of the insoluble gas in the bubble, m, is known. Find the cubic equation which must be solved to find the equilibrium radius, R, of a bubble under these conditions. [In addition to p_∞, p_V, m, ρ_L, and S the equation contains the temperature, T_B, of the bubble and the gas constant, $\mathcal{R}$, of the insoluble gas.]

[2] By considering a small departure from this equilibrium size find the inequality which governs whether this equilibrium is stable or unstable [contains R, $\mathcal{R}$, m, T_B and S].