Problem 312B

Heat is being added to the steady, frictionless flow of a perfect gas (ratio of specific heats, γ) in a pipe of constant, uniform cross-sectional area. The velocity, temperature, speed of sound and Mach number of the flow are denoted by u, T, c and M respectively and vary with position, x, measured along the pipe. The speed of sound, c, is given by $(\gamma R T)^{\frac{1}{2}}$ and the Mach number is defined as $M = u/c$.

Find the relation between the derivative $d(M^2)/dx$ and the derivative du/dx. The expression also contains c, M and γ.

If the rate of heat addition is Q per unit time per unit length of the pipe and the mass flow rate of gas is denoted by m find an expression for $d(M^2)/dx$ in terms of Q, m, γ, c and M.