Problem 295C

Suppose that, in a frame of reference fixed in a whale, the motion of the tail (the "fluke") of the whale can be described as follows:

1. An oncoming uniform stream of velocity, U, representing the forward speed of the whale (in the direction, x).
2. A "heave" motion of the fluke in the direction, y, perpendicular to the x direction such that the center of the fluke moves according to
$$y = h \sin \omega t \tag{1}$$
where ω is the frequency of the motion.
3. That the inclination of the fluke (assumed to be a flat plate) relative to the x direction is α where
$$\alpha = -\tilde{\alpha} \cos \omega t \tag{2}$$

Furthermore, assume that the motion of the fluke produces no drag force. It produces a lift force perpendicular to the instantaneous direction of relative motion equal to $\frac{1}{2}\rho V^2 c C_L$ where V is the instantaneous relative velocity, c is the chord of the fluke, ρ is the fluid density and C_L is the lift coefficient. Assume $C_L = 2\pi \sin \alpha^*$ where α^* is the instantaneous angle of attack of the relative motion.

Obtain an expression for the instantaneous lift force on the fluke non-dimensionalized by $\pi \rho c U^2$ in terms of the "reduced frequency", $\omega h/U$, the angle $\tilde{\alpha}$ and the dimensionless time, ωt.

Explain why this lift force produces a non-zero time-averaged component in the x-direction.

Using a computer program and, say, 100 time intervals in one period of oscillation find the average propulsive force in the direction of motion (x direction) non-dimensionalized by $\pi \rho c U^2$. Perform the calculation for several values of $\omega h/U$ and $\tilde{\alpha}$ to determine how the results depend on these parameters.