Problem 276A

Consider a turbulent boundary layer on a flat plate (constant and uniform velocity and pressure in the flow outside the boundary layer). The plate is very rough, the size of the roughnesses, ϵ, being very much greater than the laminar sub-layer thickness which would occur in the absence of the roughness. It is anticipated that the velocity distribution within the turbulent part of the boundary layer can be approximated by

$$u^* = C(y/\epsilon)^{\frac{1}{7}}$$

where C is some constant, y is the distance from the wall, $\bar{u}$ is the mean velocity and $u^* = \bar{u}/u_\tau$ where the friction velocity, $u_\tau = (\tau_w/\rho)^{\frac{1}{2}}$, τ_w being the wall shear stress and ρ the fluid density. Using approximate boundary layer methods find an expression for the boundary layer thickness, δ, as a function of x, the distance along the plate from the leading edge. Assume initial conditions $\delta = 0$ at $x = 0$; the result includes ϵ, C and the profile parameter $\alpha = 0.0972$.