Problem 272B

Consider planar Poiseuille flow between two stationary parallel plates distance H apart. A non-zero, constant pressure gradient, $(-dp/dx)$, creates a flow in the direction, x, parallel to the plates. If the pressure gradient in the direction, y, normal to the plates is assumed to be zero, use the momentum thereom to show that the shear stress, σ_{xy}, must vary linearly with distance, y, irrespective of the type of flow (laminar or turbulent).

If the origin of y is midway between the parallel walls show that $\sigma_{xy} = Ay$ and determine A.

Now consider that the core of this flow is turbulent. Find the hypothetical velocity profile, $\bar{u}(y)$, under the following assumptions:

- (a) The mixing length, ℓ, is constant and equal to $H/4$.
- (b) The laminar, viscous contribution to the shear stress is negligible.
- (c) The presence of the laminar sub-layers can be ignored.

[Hint: Solve for $y > 0$ first and then extend the answer to $y < 0$ in order to avoid an awkward choice-of-sign problem.]

[Note: Since the above are not very good assumptions the answer is not particularly useful but the method is similar to that which can be used with more realistic conditions.]