Problem 222A

A turbojet engine in a wind tunnel ingests air at a velocity of $100m/s$ and a density of $1kg/m^3$. The velocity is uniform and the cross-sectional area of the approaching stream which enters the engine is $0.1m^2$. The velocity of the exhaust jet from the engine, however, is not uniform but has a velocity which varies over the cross-section according to

$$u(r) = 2U \left\{ 1 - \frac{r^2}{r_0^2} \right\} \tag{1}$$

where the constant $U = 600m/s$ and r_0 is the radius of the jet cross-section. Radial position within the axisymmetric jet is denoted by r. The density of the exhaust jet is uniformly $0.5kg/m^3$.

(a) Show that U is the average velocity of the exhaust jet.

(b) Find the thrust of the turbojet engine.

(c) Find what the thrust would be if the exhaust jet had a uniform velocity, U.

Assume the pressures in both the inlet and exhaust jets are the same as the surrounding air and that mass is conserved in the flow through the engine (roughly true in practice).