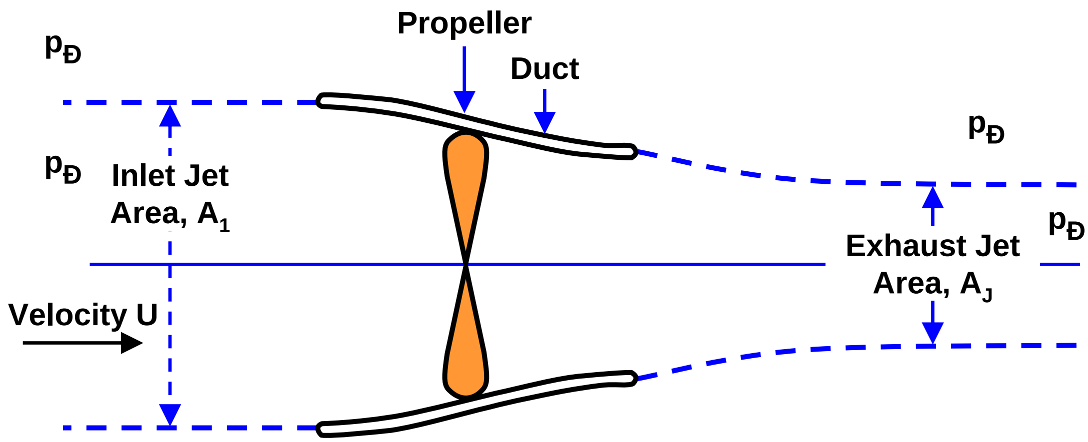
Problem 220C

A ducted propeller is used to propel an underwater vehicle which is travelling through still water at a steady velocity, U. Relative to the propeller a stream of water whose pressure is the same as the surrounding fluid, p_∞ and whose cross-sectional area is A_1, enters the duct:



The characteristic of the propeller is such that it produces a total head rise, ΔH, which is related to the volume flow rate, Q, by

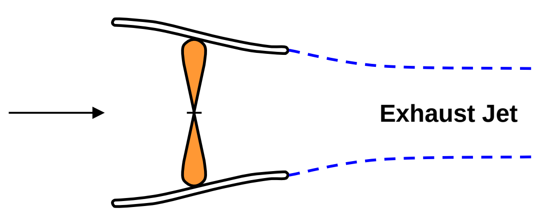
$$\Delta H = \frac{K_1 - K_2 Q}{g}$$

where K_1, K_2 are known constants and g is the acceleration due to gravity. The exhaust jet, after emerging from the duct, requires some distance before it adjusts itself so that the pressure in the jet is the same as the surrounding fluid, p_∞. The cross-sectional area of the exhaust jet after this adjustment is denoted by A_J.

Derive an expression for A_J in terms of U, K_1, K_2 and A_1 assuming that the water is inviscid and incompressible (density, ρ). What is the thrust produced by this propulsion device in terms of U, K_1, K_2, ρ and A_1? Neglect the effects of gravity.

Footnotes:

- One point of possible confusion requires clarification and physical explanation. Though it is necessary to use Bernoulli's equation in the solution, you can **not** use it directly to connect the upstream and downstream jet velocities by considering the connection **external** to the duct. Such a connection would demand that the exhaust jet velocity be U since the pressures are the same. In fact a "vortex sheet" (indicated by the dashed lines below) forms the boundary of the exhaust jet:



- Pressures are the same on either side of a vortex sheet but there is a jump in the Bernoulli constant across it.