Problem 150O

A semi-infinite domain of fluid is bounded only by a single infinite flat plate. The fluid is incompressible with a constant and uniform viscosity, μ, and density, ρ. The plate is then set in accelerating motion in its own plane with an accelerating velocity, Ue^{kt}, where U and k are constants and t is time. If the fluid only reacts by moving with a velocity, $u(y,t)$, where y is the distance from the plate and if the velocities in the other directions are zero, write down the simplified form of the Navier-Stokes equation that governs this flow and must be solved for $u(y,t)$. Note that p is uniform; that the velocity far from the plate is zero; and neglect gravitational effects. The result is a differential equation for $u(y,t)$ and only includes u, y, t and μ/ρ where μ is the viscosity of the fluid.

Using separation of variables (or otherwise) solve this equation to find $u(y,t)$ and the vorticity $\omega(y,t)$ in terms of y, t, U, k and the fluid properties. If we define a boundary layer next to the plate as the region in which the velocity is at least 10% of U, derive an expression for the thickness of the boundary layer as a function of time.