Problem 140B

Consider a flow in which the density, ρ, of a particular fluid element remains unchanged as it moves along in the flow but in which the density may vary from one fluid element to another. The fluid will be assumed to be inviscid and the body forces are assumed to be conservative.

(A) What are the forms of the equation of continuity and the equation of motion which are appropriate for such a flow ?

(B) Now consider such a flow which is just in the process of accelerating from rest at time, $t = 0$. The velocity $\underline{u}$ is zero but the acceleration, $\partial \underline{u}/\partial t$, is not zero. Show that the rate of increase of vorticity, $\underline{\omega}$, in the flow at time $t = 0$ is directly related to the density gradient and the acceleration by

$$\left(\frac{\partial \underline{\omega}}{\partial t}\right)_{t=0} = \frac{1}{\rho}\left[(\bigtriangledown \rho) \times \frac{\partial \underline{u}}{\partial t}\right]_{t=0}$$

Note the following vector identities:

$$\bigtriangledown \times (\rho \underline{a}) = (\bigtriangledown \rho) \times \underline{a} + \rho \bigtriangledown \times \underline{a} \qquad \text{where } \rho \text{ is a scalar and } \underline{a} \text{ is a vector.}$$

$$\bigtriangledown \times (\bigtriangledown y) = 0 \qquad \text{where } y \text{ is a scalar.}$$