Problem 117B

The following streamfunction, ψ, for a steady, planar, incompressible flow represents a solution for the flow around a cylinder of radius, r_0, with its center at the origin of a system of polar coordinates, (r, θ):

$$\psi = Ur\left(1 - \frac{r_0^2}{r^2}\right)\sin\theta$$

Here U is the velocity of the uniform stream in the direction $\theta = 0$ (the x direction) far away from the cylinder. [Ignore the fact that the no-slip condition is not satisfied on the surface of the cylinder.]

(a) Find the vorticity in the flow as a function of r and θ.

(b) Find the rate of deformation, e_{xy}, as a function of r and θ.

(c) Find an expression for the pressure, p, in the flow as a function of r and θ. Assume that the pressure far from the origin is p_∞ and that the body force due to gravity can be neglected.

Note: In polar coordinates, the velocities in the r and θ directions, denoted respectively by u_r and u_θ, are given by

$$u_r = \frac{1}{r}\frac{\partial\psi}{\partial\theta} \quad ; \quad u_\theta = -\frac{\partial\psi}{\partial r}$$