

### Problem 115D

Consider the following streamfunction,  $\psi$ , for a planar incompressible flow:

$$\psi = Ur \left( 1 - \frac{r_0^2}{r^2} \right) \sin \theta$$

where  $U$  and  $r_0$  are constants and  $r, \theta$  are polar coordinates.

- (a) Find and sketch the streamline corresponding to  $r = r_0$ .
- (b) Find and add to your sketch the streamlines for  $\theta = 0, r > r_0$  and for  $\theta = \pi, r > r_0$ . Note on your sketch the value of  $\psi$  along these lines and along the streamline for  $r = r_0$ .
- (c) Make a rough estimate of some other streamlines with  $\psi > 0$  and show the form of these streamlines in your sketch.
- (d) What is the magnitude and direction of the flow for  $r \gg r_0$ ?
- (e) Guided by your sketch, estimate what real flow might have the above streamfunction.

Note: In polar coordinates, the velocities in the  $r$  and  $\theta$  directions, denoted respectively by  $u_r$  and  $u_\theta$ , are given by

$$u_r = \frac{1}{r} \frac{\partial \psi}{\partial \theta} \quad ; \quad u_\theta = -\frac{\partial \psi}{\partial r}$$