Quasistatic Stability

Using the definitions of the last section, a quasistatic analysis of the stability of the equilibrium operating point is usually conducted in the following way. We consider perturbing the system to a new mass flow

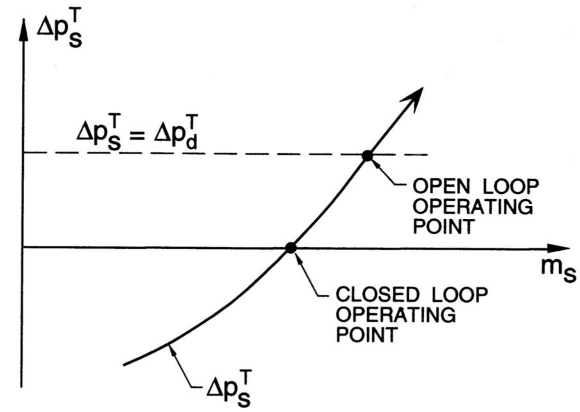

Figure 1: Typical system characteristic, $\Delta p_s^T(\dot{m}_s)$, and operating point.

rate $d\dot{m}$ greater than that at the operating point as shown in figure 2. Then, somewhat heuristically, one argues from figure 2 that the total pressure rise across the *pumping* component is now less than the total pressure drop across the *pipeline* and therefore the flow rate will decline back to its value at the operating point. Consequently, the particular relationship of the characteristics in figure 2 implies a stable operating point. If, however, the slopes of the two components are reversed (for example, Pump B of figure 3(a) or the operating point C of figure 3(b)) then the operating point is unstable since the increase in the flow has resulted in a *pump* total pressure that now exceeds the total pressure drop in the *pipeline*. These arguments lead to the conclusion that the operating point is stable when the slope of the system characteristic at the operating point (figure 1) is positive or

$$\frac{d\Delta p_s^T}{d\dot{m}_s} > 0 \quad \textbf{or} \quad R_s^* > 0 \tag{Nrc1}$$

The same criterion can be derived in a somewhat more rigorous way by using an energy argument. Note that the net flux of flow energy out of each component is $\dot{m}_k \Delta p_k^T$. In a straight pipe this energy is converted

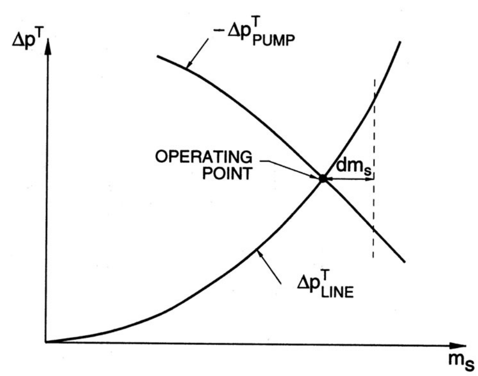

Figure 2: Alternate presentation of figure 1.

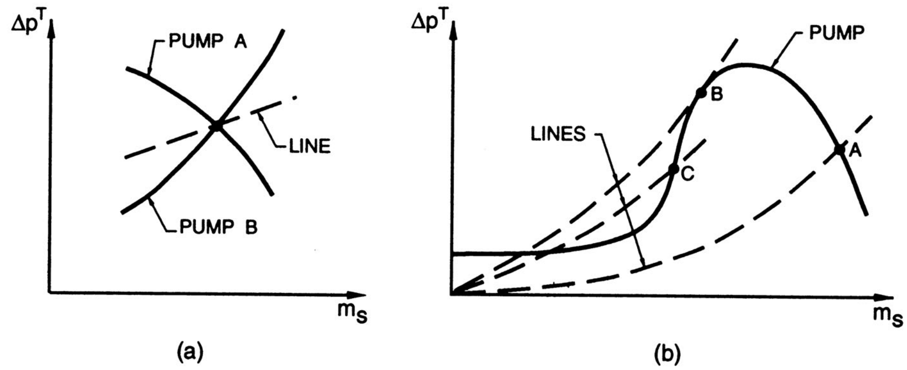

Figure 3: Quasistatically stable and unstable flow systems.

to heat through the action of viscosity. In a pump $\dot{m}_k(-\Delta p_k^T)$ is the work done on the flow by the pump impeller. Thus the net energy flux out of the whole system is $\dot{m}_s \Delta p_s^T$ and, at the operating point, this is zero (for simplicity we discuss a closed loop system) since $\Delta p_s^T = 0$. Now, suppose, that the flow rate is perturbed by an amount $d\dot{m}_s$. Then, the new net energy flux out of the system is ΔE where

$$\Delta E = (\dot{m}_s + d\dot{m}_s) \left\{ \Delta p_s^T + d\dot{m}_s \frac{d\Delta p_s^T}{d\dot{m}_s} \right\} \approx \dot{m}_s d\dot{m}_s \frac{d\Delta p_s^T}{d\dot{m}_s} \tag{Nrc2}$$

Then we argue that if $d\dot{m}_s$ is positive and the perturbed system therefore dissipates more energy, then it must be stable. Under those circumstances one would have to add to the system a device that injected more energy into the system so as to sustain operation at the perturbed state. Hence the criterion (Nrc1) for quasistatic stability is reproduced.