Two-dimensional Shocks

Noting that all of the above analyses are for simple one-dimensional flow, we should add a footnote on the nature of kinematic waves and shocks in a more general three-dimensional flow. Though the most general analysis is quite complex, a relative simple extension of the results of the preceding sections is obtained when attention is restricted to those flows in which the direction of the relative velocity vector, u_{ABi}, is everywhere the same. Such would be the case, for example, for a relative velocity caused by buoyancy alone. Let that be the 1 direction (so that $u_{AB2} = 0$) and consider, therefore, a planar flow in the 12 plane in which, as depicted in figure 1, the kinematic wave or shock is inclined at an angle, θ, to the 2 direction and is moving at a velocity, q_s, normal to itself. It is readily shown that the volume flux of any component,

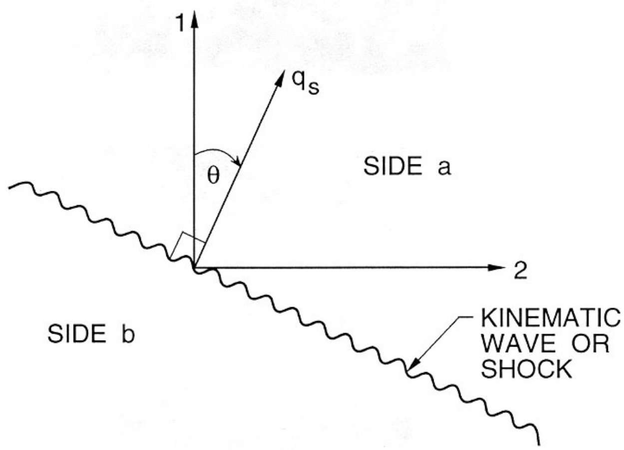

Figure 1: A two-dimensional kinematic wave or shock.

N, normal to and relative to the shock is

$$\alpha_N(u_{N2}\sin\theta + u_{N1}\cos\theta - q_s) \tag{Nsj1}$$

and the total volume flux, j_θ, relative to the shock is

$$j_\theta = j_2\sin\theta + j_1\cos\theta - q_s \tag{Nsj2}$$

Now consider a multiphase flow consisting of two components A and B, with velocity vectors, u_{Ai}, u_{Bi}, with volume flux vectors, j_{Ai}, j_{Bi}, with volume fractions, $\alpha_A = \alpha$, $\alpha_B = 1-\alpha$, and with a drift flux vector, u_{ABi} where, in the present case, $u_{AB2} = 0$. If the indices a and b denote conditions on the two sides of the shock, and if the individual volume fluxes into and out of the shock are equated, we obtain, two relations. The first relation simply states that the total volume flux, j_θ, must be the same on the two sides of the shock. The second relation yields:

$$q_s = j_\theta + \cos\theta\left\{\frac{j_{AB1a} - j_{AB1b}}{\alpha_a - \alpha_b}\right\} \tag{Nsj3}$$

which, in the case of a infinitesimal wave, becomes

$$q_s = j_\theta + \cos\theta\left.\frac{dj_{AB1}}{d\alpha}\right|_\alpha \tag{Nsj4}$$

These are essentially the same as the one-dimensional results, equations (Nse4) and (Nsb7), except for the $\cos\theta$. Consequently, within the restricted class of flows considered here, the propagation and evolution of a kinematic wave or shock in two- and three- dimensions can be predicted if the drift flux function, $j_{AB1}(\alpha)$, is known and its direction is uniform.