Sonic Speeds at Higher Frequencies

Several phenomena can lead to dispersion, that is to say to an acoustic velocity that is a function of frequency. Among these are the effects of bubble dynamics discussed in the next chapter. Another is the change that occurs at higher frequencies as the wavelength is no longer effectively infinite relative to the size of the particles. Some experimental data on the effect of the ratio of particle size to wavelength (or κR) was presented in figure 1, section (Nlb). Note that the minimum in the acoustic velocity at intermediate volume fractions disappears at higher frequencies. Atkinson and Kytömaa (1992) modeled the dynamics at non-zero values of κR using the following set of governing equations: (a) continuity equations (Nbb2) for both the disperse and continuous phases with no mass exchange ($\mathcal{I}_N = 0$) (b) momentum equations (Nbe8) for both phases with no gravity terms and no deviatoric stresses $\sigma^D_{Cki} = 0$ and (c) a particle force, F_k (see equation (Nbf4)) that includes the forces on each particle due to the pressure gradient in the continuous phase, the added mass, the Stokes drag and the Basset memory terms (see section (Neh), equation (Neh17)). They included a solids fraction dependence in the added mass. The resulting dispersion relation yields sound speeds that depend on frequency, ω, and Reynolds number, $\rho_C \omega R^2 / \mu_C$, but asymptote to constant values at both high and low Reynolds numbers. Typical results are plotted in figure **??** for various κR and exhibit fair agreement with the experimental measurements.

Atkinson and Kytömaa (1992) also compare measured and calculated acoustic attenuation rates given non-dimensionally by ζR where the amplitude decays with distance, s, according to $e^{-\zeta s}$. The attenuation results from viscous effects on the relative motion between the particles and the continuous fluid phase. At low frequencies the relative motion and therefore the attenuation is dominated by contribution from the Stokes drag term in equation (Neh17); this term is proportional to ω^2. Though the measured data on attenuation is quite scattered, the theory yields values of the dimensionless attenuation, ζR, that are roughly of the correct magnitude as shown by the example in figure 1. On the other hand at high frequencies (large κR) the theoretical attenuation is dominated by the Basset term and is proportional to $(\mu_C \omega)^{\frac{1}{2}}$; it also increases nearly linearly with the solids fraction. However the measured attenuation rates in this frequency range appear to be about an order of magnitude larger than those calculated.
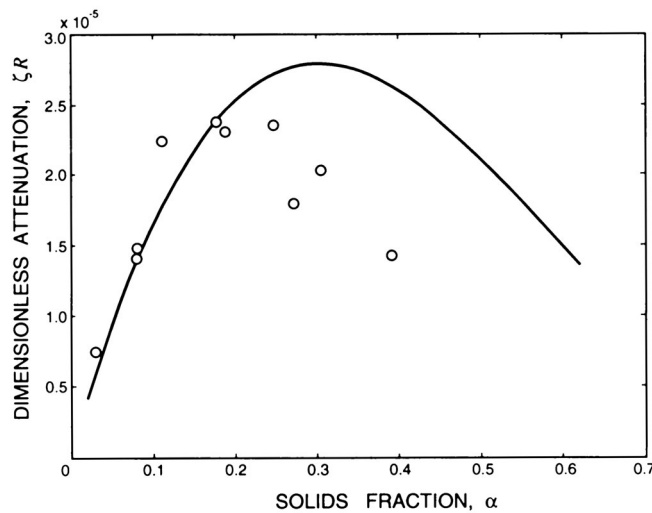

Figure 1: An example of the dimensionless attenuation, ζR, at low frequencies as a function of solids fraction, α. The experimental data ($\circ$) is for a suspension of Kaolin particles in water with $2\kappa R = 3.4 \times 10^{-4}$ (Urick 1948); the theoretical line is from Atkinson and Kytömaa (1992).

Weir (2001), following on the work of Gregor and Rumpf (1975), uses a similar perturbation analysis with somewhat different basic equations to generate dispersion relations as a function of frequency and volume fraction. Acknowledging that solutions of this dispersion relation yield a number of propagation velocities including both kinematic and dynamic wave speeds (see section (Nrm)), Weir chooses to focus on the dynamic or acoustic waves. He demonstrates that, in general, there are two types of dynamic wave. These have the same kinds of high and low frequency asymptotes described above. The two low frequency wave speeds converge to yield a single dynamic wave speed that has a functional dependence on frequency and α that is qualitatively similar to that of Atkinson and Kytömaa (1992). It also agrees well with the measured sound speeds in Musmarra *et al.*(1995) for suspensions of various types of particles in liquid. Weir also analyzes the wave speeds in fluidized beds and compares them with those in unfluidized or static beds. He also examines the data on wave attenuation; as with the other attenuation data the experimental measurements are quite scattered and do not agree well with the theoretical predictions, particularly at high frequencies.