Flow with Phase Change

The Lockhart-Martinelli correlation was extended by Martinelli and Nelson (1948) to include the effects of phase change. Since the individual mass fluxes are then changing as one moves down the pipe, it becomes convenient to use a different non-dimensional pressure gradient

$$\phi_{L0}^2 = \frac{\left(\frac{dp}{ds}\right)_{actual}}{\left(\frac{dp}{ds}\right)_{L0}} \tag{Nkg1}$$

where $(dp/ds)_{L0}$ is the hypothetical pressure gradient that would occur in the same pipe if a liquid flow with the same total mass flow were present. Such a definition is more practical in this case since the total mass flow is constant. It follows that ϕ_{L0}^2 is simply related to ϕ_L^2 by

$$\phi_{L0}^2 = (1-\mathcal{X})^{2-m_L}\phi_L^2 \tag{Nkg2}$$

The Martinelli-Nelson correlation uses the previously described Lockhart-Martinelli results to obtain ϕ_L^2 and, therefore, ϕ_{L0}^2 as functions of the mass quality, $\mathcal{X}$. Then the frictional component of the pressure gradient is given by

$$\left(-\frac{dp}{ds}\right)_{Frictional} = \phi_{L0}^2 \frac{2G^2\mathcal{K}_L}{\rho_L d}\left\{\frac{Gd}{\mu_L}\right\}^{-m_L} \tag{Nkg3}$$

Note that, though the other quantities in this expression for dp/ds are constant along the pipe, the quantity ϕ_{L0}^2 is necessarily a function of the mass quality, $\mathcal{X}$, and will therefore vary with s. It follows that to integrate equation (Nkg3) to find the pressure drop over a finite pipe length one must know the variation of the mass quality, $\mathcal{X}(s)$. Now, in many boilers, evaporators or condensers, the mass quality varies linearly with length, s, since

$$\frac{d\mathcal{X}}{ds} = \frac{\mathcal{Q}_\ell}{AG\mathcal{L}} \tag{Nkg4}$$

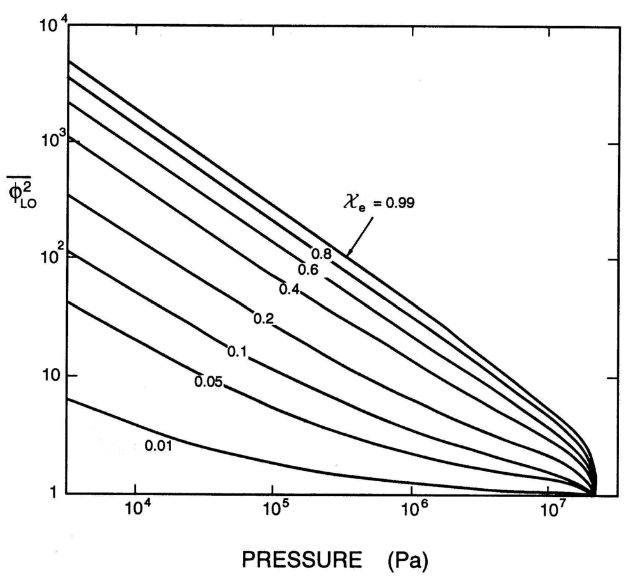

Figure 1: The Martinelli-Nelson frictional pressure drop function, $\overline{\phi_{L0}^2}$, for water as a function of the prevailing pressure level and the exit mass quality, $\mathcal{X}_e$. Case shown is for $\kappa_L = \kappa_G = 1.0$ and $m_L = m_G = 0.25$.

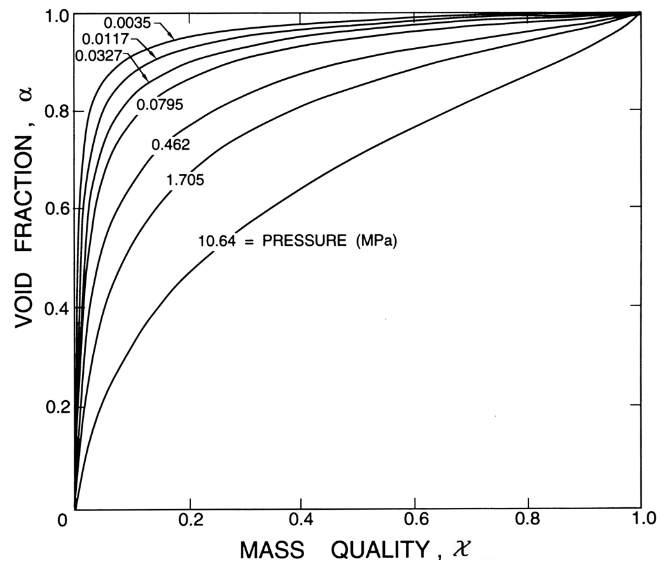

Figure 2: The exit void fraction values, α_e, corresponding to the data of figure 1. Case shown is for $\kappa_L = \kappa_G = 1.0$ and $m_L = m_G = 0.25$.

Since the rate of heat supply or removal per unit length of the pipe, $\mathcal{Q}_\ell$, is roughly uniform and the latent heat, $\mathcal{L}$, can be considered roughly constant, it follows that $d\mathcal{X}/ds$ is approximately constant. Then integration of equation (Nkg3) from the location at which $\mathcal{X} = 0$ to the location a distance, ℓ, along the pipe (at which $\mathcal{X} = \mathcal{X}_e$) yields

$$(\Delta p(\mathcal{X}_e))_{Frictional} = (p)_{\mathcal{X}=0} - (p)_{\mathcal{X}=\mathcal{X}_e} = \frac{2G^2 \ell \mathcal{K}_L}{d\rho_L} \left\{ \frac{Gd}{\mu_L} \right\}^{-m_L} \overline{\phi_{L0}^2} \qquad \text{(Nkg5)}$$

where

$$\overline{\phi_{L0}^2} = \frac{1}{\mathcal{X}_e} \int_0^{\mathcal{X}_e} \phi_{L0}^2 d\mathcal{X} \qquad \text{(Nkg6)}$$

Given a two-phase flow and assuming that the fluid properties can be estimated with reasonable accuracy by knowing the average pressure level of the flow and finding the saturated liquid and vapor densities and viscosities at that pressure, the results of the last section can be used to determine ϕ_{L0}^2 as a function of $\mathcal{X}$. Integration of this function yields the required values of $\overline{\phi_{L0}^2}$ as a function of the exit mass quality, $\mathcal{X}_e$, and the prevailing mean pressure level. Typical data for water are exhibited in figure 1 and the corresponding values of the exit void fraction, α_E, are shown in figure 2.

These non-dimensional results are used in a more general flow in the following way. If one wishes to determine the pressure drop for a flow with a non-zero inlet quality, $\mathcal{X}_i$, and an exit quality, $\mathcal{X}_e$, (or, equivalently, a given heat flux because of equation (Nkg4)) then one simply uses figure 1, first, to determine the pressure difference between the hypothetical point upstream of the inlet at which $\mathcal{X} = 0$ and the inlet and, second, to determine the difference between the same hypothetical point and the outlet of the pipe.

But, in addition, to the frictional component of the pressure gradient there is also a contribution caused by the fact that the fluids will be accelerating due to the change in the mixture density caused by the phase change. Using the mixture momentum equation (Nbe13), it is readily shown that this acceleration contribution to the pressure gradient can be written as

$$\left(-\frac{dp}{ds}\right)_{Acceleration} = G^2 \frac{d}{ds} \left\{ \frac{\mathcal{X}^2}{\rho_G \alpha} + \frac{(1-\mathcal{X})^2}{\rho_L(1-\alpha)} \right\} \qquad \text{(Nkg7)}$$

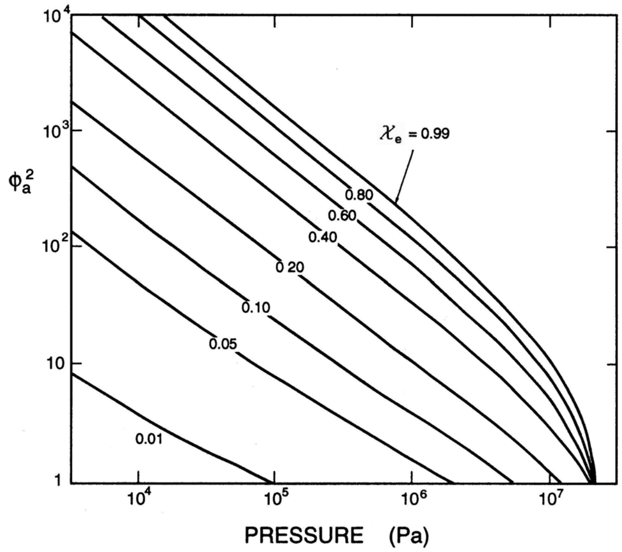

Figure 3: The Martinelli-Nelson acceleration pressure drop function, ϕ_a^2, for water as a function of the prevailing pressure level and the exit mass quality, $\mathcal{X}_e$. Case shown is for $\kappa_L = \kappa_G = 1.0$ and $m_L = m_G = 0.25$.

and this can be integrated over the same interval as was used for the frictional contribution to obtain

$$(\Delta p(\mathcal{X}_e))_{Acceleration} = G^2 \rho_L \phi_a^2(\mathcal{X}_e) \tag{Nkg8}$$

where

$$\phi_a^2(\mathcal{X}_e) = \left\{ \frac{\rho_L \mathcal{X}_e^2}{\rho_G \alpha_e} + \frac{(1-\mathcal{X}_e)^2}{(1-\alpha_e)} - 1 \right\} \tag{Nkg9}$$

As in the case of $\overline{\phi_{L0}^2}$, $\phi_a^2(\mathcal{X}_e)$ can readily be calculated for a particular fluid given the prevailing pressure. Typical values for water are presented in figure 3. This figure is used in a manner analogous to figure 1 so that, taken together, they allow prediction of both the frictional and acceleration components of the pressure drop in a two-phase pipe flow with phase change.