

Cavitation Inception

It is conventional to characterize how close the pressure in the liquid flow is to the vapor pressure (and therefore the potential for cavitation) by means of the *cavitation number*, σ , defined by

$$\sigma = \frac{p_\infty - p_V(T_\infty)}{\frac{1}{2}\rho_L U_\infty^2} \quad (\text{Nhb1})$$

where U_∞ , p_∞ and T_∞ are respectively a reference velocity, pressure and temperature in the flow (usually upstream quantities), ρ_L is the liquid density and $p_V(T_\infty)$ is the saturated vapor pressure. In a particular flow as σ is reduced, cavitation will first be observed to occur at some particular value of σ called the incipient cavitation number and denoted by σ_i . Further reduction in σ below σ_i would cause an increase in the number and size of the vapor bubbles.

Suppose that prior to cavitation inception, the magnitude of the lowest pressure in the single phase flow is given by the minimum value of the coefficient of pressure, C_{pmin} . Note that C_{pmin} is a *negative* number and that its value could be estimated from either experiments on or calculations of the single phase flow. Then, *if cavitation inception were to occur when the minimum pressure reaches the vapor pressure* it would follow that the value of the critical inception number, σ_i , would be simply given by

$$\sigma_i = -C_{pmin} \quad (\text{Nhb2})$$

Unfortunately, many factors can cause the actual values of σ_i to depart radically from $-C_{pmin}$ and much research has been conducted to explore these departures because of the importance of determining σ_i accurately. Among the important factors are

1. the ability of the liquid to sustain a tension so that bubbles do not grow to observable size until the pressure falls a finite amount below the vapor pressure. The magnitude of this tension is a function of the contamination of the liquid and, in particular, the size and properties of the microscopic bubbles (*cavitation nuclei*) that grow to produce the observable vapor bubbles (see, for example, Billet 1985).
2. the fact the cavitation nuclei require a finite residence time in which to grow to observable size.
3. the fact that measurements or calculations usually yield a minimum coefficient of pressure that is a time-averaged value. On the other hand many of the flows with which one must deal in practice are turbulent and, therefore, nuclei in the middle of turbulent eddies may experience pressures below the vapor pressure even when the mean pressure is greater than the vapor pressure.

Moreover, since water tunnel experiments designed to measure σ_i are often carried out at considerably reduced scale, it is also critical to know how to scale up these effects to accurately anticipate inception at the full scale. A detailed examination of these effects is beyond the scope of this text and the reader is referred to Knapp, Daily and Hammitt (1970), Acosta and Parkin (1975), Arakeri (1979) and Brennen (1995) for further discussion.

The stability phenomenon described in section (Nge) has important consequences in many cavitating flows. To recognize this, one must visualize a spectrum of sizes of cavitation nuclei being convected into a region of low pressure within the flow. Then the p_∞ in equations (Nge1) and (Nge7) will be the local pressure in the liquid surrounding the bubble, and p_∞ must be less than p_V for explosive cavitation growth to occur. It is clear from the above analysis that all of the nuclei whose size, R , is greater than some critical value

will become unstable, grow explosively, and cavitate, whereas those nuclei smaller than that critical size will react passively and will therefore not become visible to the eye. Though the actual response of the bubble is dynamic and p_∞ is changing continuously, we can nevertheless anticipate that the critical nuclei size will be given approximately by $4S/3(p_V - p_\infty)^*$ where $(p_V - p_\infty)^*$ is some representative measure of the tension in the low-pressure region. Note that the lower the pressure level, p_∞ , the smaller the critical size and the larger the number of nuclei that are activated. This accounts for the increase in the number of bubbles observed in a cavitating flow as the pressure is reduced.

It will be useful to develop an estimate of the maximum size to which a cavitation bubble grows during its trajectory through a region where the pressure is below the vapor pressure. In a typical external flow around a body characterized by the dimension, ℓ , it follows from equation ?? that the rate of growth is roughly given by

$$\frac{dR}{dt} = U_\infty(-\sigma - C_{pmin})^{\frac{1}{2}} \quad (\text{Nhb3})$$

It should be emphasized that equation (Ngd8) implies explosive growth of the bubble, in which the volume displacement is increasing like t^3 .

To obtain an estimate of the maximum size to which the cavitation bubble grows, R_m , a measure of the time it spends below vapor pressure is needed. Assuming that the pressure distribution near the minimum pressure point is roughly parabolic (see Brennen 1995) the length of the region below vapor pressure will be proportional to $\ell(-\sigma - C_{pmin})^{\frac{1}{2}}$ and therefore the time spent in that region will be the same quantity divided by U_∞ . The result is that an estimate of maximum size, R_m , is

$$R_m \approx 2\ell(-\sigma - C_{pmin}) \quad (\text{Nhb4})$$

where the factor 2 comes from the more detailed analysis of Brennen (1995). Note that, whatever their initial size, all activated nuclei grow to roughly the same maximum size because both the asymptotic growth rate (equation (Ngd8)) and the time available for growth are essentially independent of the size of the original nucleus. For this reason all of the bubbles in a bubbly cavitating flow grow to roughly the same size (Brennen 1995).